SPIRITUAL STUDY GROUP
(SSG)

आध्यात्मिक अध्ययन संस्था
(आऽस)

ॐ श्रीः

# VEDIC MATHEMATICS
# OR
# SIXTEEN SIMPLE MATHEMATICAL FORMULAE FROM THE VEDAS

*(For One-line Answers to all Mathematical Problems)*

## -: वैदिक गणित सूत्र :-

१ एकाधिकेन पूर्वेण
२ निखिलं नवतश्चरमं दशतः
३ ऊर्ध्वतिर्यग्भ्याम्
४ परावर्त्य योजयेत्
५ शून्य साम्यसमुच्चये
६ (आनुरूप्ये) शून्यमन्यत्
७ संकलनव्यकलनाभ्याम्
८ पूरणपूरणाभ्याम्

९ चलनकलनाभ्याम्
१० यावदूनम्
११ व्यष्टिसष्टिः
१२ शेषाण्यङ्केन चरमेण
१३ सोपान्तयद्वयमन्त्यम्
१४ एकन्यूनेन पूर्वेण
१५ गुणितसमुच्चयः
१६ गुणकसमुच्चयः

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वेदात सर्वं प्रसिद्धयति ।

वेद सम्पूर्ण ज्ञान के अनादि अनन्त स्रोत ।

**Vedas are the Eternal Source of Knowledge !**

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गुरुकुल कांगड़ी विश्वविद्यालय
कृपया पुस्तक के ऊपर कोई निशान आदि
न लगायें।

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R पुस्तकालय
गुरुकुल कांगड़ी विश्वविद्यालय, हरिद्वार
वर्ग संख्या 510 आगत संख्या 5979
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पुस्तक—वितरण की तिथि नीचे अंकित है। इस तिथि सहित २० वें दिन तक यह पुस्तक पुस्तकालय में वापिस आ जानी चाहिए। अन्यथा १० पैसे के हिसाब से विलम्ब—दण्ड लगेगा।

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आध्यात्मिक अध्ययन संस्था (आ॰स)
वैदिक विज्ञान संस्थान

SPIRITUAL STUDY GROUP (SSG)
Institute of Vedic Sciences
41/2, Amod Kunj
University Campus
ROORKEE-247 667 (INDIA)

# वैदिक गणित अध्ययन

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SEMINAR

ON

VEDIC MATHEMATICS

From December 24 to December 26, 1987

at

DEPARTMENT OF MATHEMATICS

FACULTY OF SCIENCE

Banaras Hindu University

VARANASI

to

celebrate

Srinivasa Rramanujan Centenary

&

125 birth - day of Pandit

Mahamana Madan Mohan Malviya ji

Coorganised by the S.S. G. U.O. R.
Roorkee

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## LIST OF LECTURE

Ist day : Dec. 24, 1987

1. Dr. N. Puri – An overview of Vedic Maths.
2. Miss Uma Arora – NIKHILAM method of multiplication and corollaries.
3. Dr. N. Puri – Superfast Vadic method of Addition, subtraction, Tables, Vinculum and their checking.

IInd day Dec. 25, 1987

4. Dr. N. Puri – Use of different bases and subbases
5. Miss Pragati Guel – Nihilam and Paravartaya method of division.
6. Dr. N.Puri – Urdhava method of multiplication
7. Dr. N. Puri – Ekadhika metbod for recurring decimals and Vedic Alpha numeric code.

IIIrd day Dec. 26, 1987

8. Dr. N. Puri – Division by Dhwajhank
9. Miss Pragati Guel – Squaring, Cubing and Square root of numbers.
10. Dr. N. Puri – Determinants and solution of simultaneous equations.
11. Miss Uma Arora – Solution of ordinary differential equations.

These lectures have been delivered at the Seminar on
VEDIC MATHEMATICS
at
The Department of Mathematics,
Banaras Hindu University
Varanasi

Teachers :

1) Dr. N. Puri
Secretary
University Campus
S.S.G. Roorkee

2) Miss Uma Arora,
Member
Vedic Maths ,
Research Group,
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Vedic Maths,
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University Campus, Roorkee

आध्यात्मिक अध्ययन संस्था (आअस)
वैदिक विज्ञान संस्थान

Spiritual Study Group (SSG)
INSTITUTE OF VEDIC SCIENCES
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# वैदिक गणित अध्ययन

WORKSHOP
ON
VEDIC MATHEMATICS

held at
Mathematics Department
BANARAS HINDU UNIVERSITY
from 24-26 Dec., 1987

Lecture Notes of
Lectures delivered
by
Member of
Vedic Mathematics Research Group
ROORKEE

**वेदात् सर्वं प्रसिद्ध्यति**
**वेद सम्पूर्ण ज्ञान के अनादि अनन्त स्रोत है।**
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## OPINIONS OF THE PARTICIPANTS:

Everybody who has learnt a little about the Vedic Mathematics has been thrilled by the superfast, almost magic speed calculation procedures of the Vedic Mathematics.

The Spiritual Study Group at Roorkee has been organising, Elementary and Advanced Courses on Vedic Mathematics since May, 85. All the participants are asked to fill up the response forms comparing the present system of Mathematics with the Vedic system. A total of 11 questions were asked. The minimum score in favour of Vedic Mathematics for any question in any course sofar was 80%. Must of the questions got 100% favour. Further, comments of participants are motivating and inspiring for spreading Vedic Mathematics

A few glimpses of the participants opinions :

- A boon for all competitive examinations.
- Could never imagine it would be so interesting.
- Very pleasant to have this course at the university.
- How advanced our ancestors were even in Mathematics.
- If Vedic Maths was taught in every School, computers would have developed in India.
- Excellent course.
- Want to learn advanced topics.
- Increases devotion towards our INDIA and its ancient culture.
- Now I wish to learn Sanskrit to know more about our ancient Scientific developments.
- Every person on earth should have the knowledge of Vedic Mathematics.
- It was a boon to learn Vedic Mathematics in Dushera holidays ( I had planned for my regular studies ).
- Want to learn more about Vedic Mathematics like factorisation etc, which are difficult in modern system.
- Amazing speed, very wide applications covering almost all the mathematical operations.
- A dynamic step for its introduction in educational system is a must.
- Has created an interest to know more about it.
- It can be of great use for children if made part of sysyllabus.

Contd.......2/-

- Mysterious nature of numbers. After little practice problems can be mentally solved at magic speed.
- Very educative, has much scope for coming generation to start its practice from beginning.
- It is a strong proof of our rich cultural heritage.
- By Vedic methods maths which is a burdoh, be-comes a joy for the children.
- This is the most interesting and thrilling time of my life. Being a mathematics teacher I have interest in maths but this Vedic Mathematics has thrilled me. When I came in contact with the Vedic Mathematics it gave such a pleasure which I can't explain in words.

A teacher of Adarsh Bal Niketan

- The lectureswere eye openers and most thrilling
- A glimpse of the lofty trakking in Himalayas of ancient knowledge in Maths, developed with spiritualism. Cudos to SSG.

A Professor of University

THE TIMES OF INDIA, NEW DELHI FRIDAY, NOVEMBER 27, 1987

Project to study use of Vedic Maths in Computers

New Delhi, November 26 ( UNI) The recently set-up Rashtriya Ved Vidya Pratishthan ( National foundation for Vedic studies ) has been asked to prepare a project to examine the applicability of ancient Vedic Mathematics to modern computer calculations, the human resource development minister, Mr. P.V. Narasimha Rao, told the Lok Sabha today.

Replying to supplementaries by Dr. A. K. Patel, he said this was decided after researchers at the California university found out that Indian mathematicians of the Vedic period knew and successfully employed the binary and other numbers which formed the basis of the modern computer concept.

This news has also appeared in

1. PATRIKA, Kanpur Friday Nov.27, 1987 issue Ancient Capability in computer calrulations.
2. Aaj - Kanpur Nov. 27, 1987 ( Hindi )
3. Navjeevan Lucknow Nov. 27, 1987 ( Hindi )
4. Nav bharat New Delhi, Nov. 27, 1987 ( Hindi )
5. National Herald Nov. 27, 1987 ( English )
6. Hindustan Delhi Nov. 27, 1987 ( Hindi )
7. Detailed report was Published in The Hindustan Times, Nov, 27 1987 (E).

आध्यात्मिक अध्ययन संस्था (आऽस)
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# वैदिक गणित अध्ययन

## WORKSHOP ON ELEMENTARY VEDIC MATHS

Held at Mathematics Department
Banaras HinduUniversity, VARANASI

DEC.24-26, 1987

VEDIC MATHS

Elementary

| S.No. | Chapter No. | Title |
|---|---|---|
| 1, | 1. | Base System |
| 2. | 2. | Tables |
| 3. | 3. | Vinculum |
| 4. | 24. | Vinculum Operations |
| 5. | 26. | Mixed Addition and Subtraction |
| 6. | 13. | Multiplication general method |
| 7. | 14. | Multiplication check |
| 8. | 20 | Compound multiplication, squares and Cubic Measures |
| 9. | 32 | Straight division |
| 10. | 33 | Division check |
| | Higher VM | |
| 11. | II-1. | Squaring-Urdhava and Yaradunam Sutra |
| 12. | II-2. | Squaring - Dwandwa Yoga |
| 13. | II-3. | Square Rooting |
| 14. | 8 | Evaluation of Determinants |
| 15 | 10 | Change of Roots of Equation |
| 16 | 11 | Linear and Non linear Differential Equations |

Teachers

1. Dr. Narender Puri
2. Ms. Pragati Goel
3. Ms. Uma Arora

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# CHAPTER-1

## BASE SYSTEM

In Vedic Mathematics the base system is very frequently used to reduce the computations involved. The base is considered to be the powers of 10. For example 10, 100, 1000, 10,000 and so on are the different bases.

Deviation from the Base:

1. We should take, as base, for our calculations, that power of 10 which is nearest to the number.
2. For the numbers more than the base, the portion of the number exceeding the base is written with a +ve sign in front of the number.
3. For the number below the base, we substract the number from the base and write down the remainder in front of the number with a -ve sign. This deviation is directly written by using the Nikhilam Sutra. (Sutra No.2). The Sutra means All from 9 and last from 10'.

Examples:

1.1 Say Number is 103
The base is 100 and the number is 3 more than the base.
Therefore, 103 is written as 103+03

1.2 Say Number is 115
The base is 100 and the number is 15 more than the base.
Therefore, 115 is written as: 115+15

1.3 Say Number is 1012
Now base is 1000 and the number is 12 more than the base.
Therefore, 1012 is written as: 1012+012

1.4 Say Number is 13
This time base is 10 and the number is 3 more than the base
Therefore, 13 is written as: 13+3

1.5 Say Number is 10004
This time base is...... and the number is .......
more than the base.
Therefore ........ is written as ...: ...+ .....

1.6 Say the Number is 9
The base is 10 and 10 - 9 gives a remainder of 1,
therefore 9 is one below the base 10.
The number 9 is written as: 9 - 1

1.7 Say the number is 92
The base is 100 and the number is 8 less than the base
Therefore 92 is written as: 92 - 08

1.8 Say the number is 991.
The base is 1000 and the number is 9 below than the base.
Therefore, 991 is written as: 991 -009
By Nikhilam Sutra we get 9-9=0.9-9=0and 10-1=9
Therefore deviation = - 009.

1.9 Say the Number is 75.
The base is 100 and the number is 25 less than the base.
Therefore, 75 is written as: 72-25

1.10 Say the Number is 971
The base is 1000 and the Number is 29 less than the base
Therefore, 971 is written as: 971-029
By Nikhilam Sutra we get 9-9=0; 9 - 7 = 2; and 10-1=9
Therefore deviation = -029.

1.11 Say the number is 9975.
The base is ..... and the number is ....... less than the base.
Therefore ....... is written as: ..........

Note: If the difference of the number from the base is very large, a suitable sub-base (or working base) can also be used. The sub-base is a convenient multiple of a suitable base. For example the number is 41. If we take a base of 10, it is 31 more than 10. If we take a base of 100, it is 59 less than 100. Therefore, it shall not be convenient to use a base of 10 or 100 and a working base shall have to be used. (To be explained later on, in subsequent chapters.).

## CHAPTER-2

## TABLES

### 2.1 Introduction :

Nikhilam Sutra - This is the second sutra. Using this sutra we can very conveniently perform multiplication of numbers consisting of very large or small digits. In this process the Vedic Base System is used.

The simple meaning of this sutra is All from 9 and last from 10. This sutra is used to straight away write down the deviation of the numbers from the base for the numbers smaller than the base. Let us take an example.

Illustration 1 : Say the number is 9856. The base (i.e. the multiple of 10) close to the number is 10000. The Nikhilam sutra which reads Nikhilam Navtashcaraman Dashatah tell us to subtract each digit of the number from 9 and the last digit from 10.

Therefore for given number 9856 the digit by digit deviation is obtained as

9 - 9 0, 9 - 8 = 1, 9 - 5 = 4 and the last digit of the deviation is 10-6 = 4. Hence the deviation is -0144 and in the Vedic Base System we write it as

9856 - 0144

Similarly 937 is written as

937 - 063

73 is written as 73 - 27.

Illustration 2 : If the given number is larger than the base or sub-base we write down the deviation with a positive sign as already explained.

### Discussion

This is very useful for multiplying the numbers which are closer to the base or a suitable sub-base. As the process of multiplication is simply reduced to the multiplication of the deviations and simple algebraic addition. But before taking the details about the same we shall see how this approach of Vedic Mathematics makes the whole process of number tables a very easy affair. As a matter of fact we shall presently see that there is no need to remember tables beyond 5x5 as anything beyond can be simply split down to the level of 5x5. The idea is not to say that one should not remember the tables but it is not essential to mug-up the tables..

## 2.2 TABLES

The Vedic method gives us the key which works superfast not only upto tables of 20 but is equally efficiently applicable to any table even upto 100 or more.

### 2.2.1 The Operation :

Illustration 3 : Let us take the table of 19 as our first example.

The operator is -1 for the unit place and +2 for the tenth place.

(i) That means we write the number 19 and divide it into two parts by drawing a vertical line in between the two digits.

(ii) We apply the operator to the digit at the unit place (with appropriate sign -of course). In the present example we appl the operator -1 to the digit 9(at the unit place). and therefore we get 9-1=8 as the second level unit number, 8-1=7 as a third level digit, 7-1 =6 the fourth,level, 6-1=5 as the fifth level and so on, finally 1-1=0 at the tenth level. However,if required,the carry over can be easily done.

This step gives us the digits _ of the unity column

| | |
|---|---|
| 1 | 9 |
| | 8 |
| | 7 |
| | 6 |
| | 5 |
| | 4 |
| | 3 |
| | 2 |
| | 1 |
| | 0 |

(iii) Next we apply the operator or the tenth place to the digit at the tenth place. Presently we add +2 at every stage to get the tenth place number at the next level. Therefore we get 1+2=3 as the digit at the second level. Similarly moving further, 3+2=5 is the third level digit, 5+2=7 is the fourth level digit

7+2=9 the fifth level, 9+2=11 is the sixth level digit, 11+2=13 as the seventh level digit and so on, 17+2=19 at the tenth level.

Notes:

1. However it is not essential to stop at tenth level as the same operators can be further applied for continuing to write the table upto hundredth level or even more if so desired.

2. As soon as the operation of addition gives the value larger than 9, the unity at the tenth place of addition (which shall be actually at the hundredth place of the final number) we can leave in the addition process and add only to the single digit left over.

| Level | | |
|---|---|---|
| 1 | 1 | 9 |
| 2 | 3 | 8 |
| 3 | 5 | 7 |
| 4 | 7 | 6 |
| 5 | 9 | 5 |
| 6 | 11 | 4 |
| 7 | 13 | 3 |
| 8 | 15 | 2 |
| 9 | 17 | 1 |
| 10 | 19 | 0 |

Illustration 4: Before we discuss the simple step-by-step process of establishing and identifying the operator key numbers for writing the tables let us take another example of the table of 89. This number being closer to the base of 100(with three digits in the base) we require three operators. It may be surprisingly interesting to learn that the rather simple operator numbers for this table of 89 are -1, -1 and +1 for the unity, tenth and hundredth place respectively.

Step 1 : Once again we divide the number 89 into three parts by drawing two vertical lines, one between the two digits and the second before the tenth place digit.

Step 2: As before we apply an operator with appropriate sign to the unit place digit. Therefore we get 9-1=8 as the second level number, 8-1=7 as the third level etc., 1-1=0 as the tenth level. We may be interested to go even beyond tenth level. Now our unit place digit at the tenth level is 0. Therefore we simply carry 1 from the

| Level | | |
|---|---|---|
| 1 | 8 | 9 |
| 2 | | 8 |
| 3 | | 7 |
| 4 | | 66 |
| 5 | | 5 |
| 6 | | 4 |
| 7 | | 3 |
| 8 | | 2 |
| 9 | | 1 |
| 10 | | 0 |
| 11 | -1 | 9 |
| 12 | | 8 |
| 13 | | 7. |
| 18 | | 2 |
| 19 | | 1 |
| 20 | | 0 |
| 21 | | |

tenth place and get 10 (to be operated upon) . However we also record -1 infront of the level after which the carry over had to be done. (In the present example, we put -1 infront of the eleventh level). Therefore, the unit number of the eleventh level is 10-1=9, twelveth level number =9-1=8 and so on.

Step 3: Similarly we apply the operator of the tenth place to the digit at the tenth place. In the present case we subtract 1 (as our operator is -1, therefore we apply -1 at each level) from the tenth place digit to get the corresponding digit at the next level. Here we get 8-1=7 at the Second level digit etc. 1-1=0 is the ninth level digit. Once again we have to carry over unity from the hundredth place to the tenth place. Once again we record -1 in front of the next level i.e. the tenth level in the hundredth place. Therefore we get ten and the tenth level digit at the tenth place is 10-1=9, the eleventh place digit is 9-1=8, But we observe that there is already -1 at the eleventh place (due to the carry over to unit place during the previous operation). Therefor the final eleventh level digit at the tenth place is obtained by further subtracting this carry over 1 from the already obtained digit (8) = 8-1=7. Further 7-1 =6 is the twelvth level digit etc.

| Level | | | |
|---|---|---|---|
| 1 | | 8 | 9 |
| 2 | | 7 | 8 |
| 3 | | 6 | 7 |
| 4 | | 5 | 6 |
| 5 | | 4 | 5 |
| 6 | | 3 | 4 |
| 7 | | 2 | 3 |
| 8 | | 1 | 2 |
| 9 | | 0 | 1 |
| 10 | -1 | 9 | 0 |
| 11 | | 7 | 9 |
| 12 | | 6 | 8 |
| | | . | . |
| | | . | . |
| 17 | | 1 | 3 |
| 18 | | 0 | 2 |
| 19 | -1 | 9 | 1 |
| 20 | | 8 | 0 |
| 21 | | 6 | 9 |

Note: Please try to complete and get the digits at least upto twenty second level.

Step 4: Further we apply the simple operator (+1 in this case) of the hundredth place to the corresponding digit (which is 0 here). Therefore we get 0+1 = 1 the second level digit at the hundredth place, 1+1 = 2 the third level digit, 2+1=3 the fourth level digit etc. 7+1=8 at the ninth level of hundredth place, 8+1=9 is the tenth level digit. But as we try to enter this at the tenth level we find -1 recorded due to the carry over to the tenth place therefore the final digit at the tenth level shall be 1 less than the digit obtained. Hence 9-1=8 is the tenth level digit at the hundredth place etc. Please note that similar reduction by 1 shall also be required at the ninteenth level and so on.

| Level | | | | |
|---|---|---|---|---|
| 1 | | 0 | 8 | 9 |
| 2 | | 1 | 7 | 8 |
| 3 | | 2 | 6 | 7 |
| 4 | | 3 | 5 | 6 |
| 5 | | 4 | 4 | 5 |
| 6 | | 5 | 3 | 4 |
| 7 | | 6 | 2 | 3 |
| 8 | | 7 | 1 | 2 |
| 9 | | 8 | 0 | 1 |
| 10 | | 8 | 9 | 0 |
| 11 | | 9 | 7 | 9 |
| 12 | 1 | 0 | 6 | 8 |
| | | | | ⋮ |
| 17 | 1 | 5 | 1 | 3 |
| 18 | 1 | 6 | 0 | 2 |
| 19 | 1 | 6 | 9 | 1 |
| 20 | 1 | 7 | 8 | 0 |
| 21 | 1 | 8 | 6 | 9 |
| | | | | ⋮ |

Notes:

1. The two examples although have taken rather a long space in explaining the step-by-step process of applying the operators for each digit, but once understood the actual application can be straight away easily mentally performed even by an average student of IV class.

2. Further it is important to note that the operators in any case shall never be more than ± 5 and as such the tables upto 5x5 would be sufficient.

3. The operators can be conveniently written at the top of corresponding digits and the mentally computed digits at each level are directly written at the appropriate place and column.

Illustration 5 : Similarly we can easily apply the operators for any other table (once the operators have been identified). Let us take another example of table for the number 22. The operators for this table are +2 and +2 for the unit and tenth place respectivelly.

**Step 1 :** We divide the number 22 into two parts by drawing a vertical line.

**Step 2 :** We apply the operator to the digit at the unit place to get the digit at the subsequent level. Therefore 2+2=4 is the second level number, 4+2=6 is the third level number, 6+2=8 the fourth level number, 8+2=10 the fifth level number, we have got the number greater than 9 therefore 1 is carried to the tenth place and recorded at fifth level in the tenth column with the positive sign. Further 0+2=2 gives us the sixth level digit etc.

| | |
|---|---|
| 2 | 2 |
| | 4 |
| | 6 |
| | 8 |
| +1 | 0 |
| | 2 |
| | 4 |
| | 6 |

**Step 3:** We apply the operator of the tenth place to the corresponding digit. In the present example we get 2+2=4 as the tenth place digit at second level, 4+2=6 is the third level digit, 6+2=8 the fourth level digit, 8+2=10 the fifth level digit. Once again we find that there is already +1 at the fifth level due to the carry over from the unit place. Therefore the final number at the fifth level is 10+1=11. Here for subsequent levels one shall be entered uniformly in the hundredth place. Further 1+2=3 is the sixth level digit and so on.

| | |
|---|---|
| 2 | 2 |
| 4 | 4 |
| 6 | 6 |
| 8 | 8 |
| 11 | 0 |
| 13 | 2 |
| 15 | 4 |
| 17 | 6 |
| 19 | 8 |
| 22 | 0 |

### 2.2.2 The Operators

We have seen how easily we can operate each digit of the number to write down the table. The entire operation shall requi just a few seconds time. The only question left over is how do we identify the operators at the unity, tenth and hundredth plac This is very conveniently done by establishing the set of links between the number and the appropriate base through the suitable sub-base, we frequently invoke the Nikhilam Sutra for this purpos The required operators in this link chain are the desired operat

Illustration 6 : Let us study the number 19. 19 = 20-1=2x(10)-1x(1) is the link chain, between the number 19 and the unity&10. Further we identify that 10 is to be multiplied by +2 and 1 is multiplied by -1. Hence the operators for the tenth and the unit place are +2 and -1 respectively. Once the appropriate operators have been identified the simple mental operations can be performed to get the table in a matter of a few seconds.The detailed step-by-step process has been already explained in the illustrations number 3,4 and 5.

Illustration 7 : Further the 89 (illustration 3) can be linked with the base 1 and 10 as follows

89 = 90 -1 = 9x(10)-1x(1)

Therefore, we find the required operators are -1 and +9 for the unit and tenth place. However, as already mentioned, the Vedic Mathematics does not require even the operation with +9. The operators can be further simplified by establishing another link of the tenth place digit (9) with the hund.edth place as this number is closer to the base 100.

Alternatively 89 = 90-1, 90=100-10. Therefore 89= 100-10-1 = 1x(100)-1x(10)-1x(1) we find that the operators in this link chain are +1, -1 and -1 for the hundredth, tenth and unit place hence they are also the corresponding operators for the table 89. Please note that each operator is picked up alongwith its appropriate sign.

After identifying the operators the table can be written down upto any desired level, as already explained in illustration 4.

Further Illustrations:

(8) 22 = 20+2 = 2x(10)+2x(1)

Therefore +2 and +2 are the operators at the tenth and unit place respectively.

(9) 17 = 10+7=1x(10)+7x(1)

with operators of 1 and 7 but 7 is too big a number to be permissible in the Vedic System. Therefore preferably and more conveniently we can say that

17 = 20-3 = 2x(10)-3x(1) with convenient operators of +2 and -3 at the tenth and unit place respectively.

(10) 28 = 30-2=3x(10)-2x(1) give the convenient operators of +3 and -2.

(11) 34 = 30+4 = 3x(10)+4x(1)

(12) 85 = 90-5 = 9x(10)-5x(1), too big operators

∴ 85 = 90-5 = 100-10-5 = 1x(100)-1x(10)-5x(1)

(13) 97 = 100-3 = 1x(100)+0x(10)-3x(1)

with 1, 0, -3 as the three operators.

Discussion

The same system can be easily extended to even for writing the tables of big numbers. The only difference being a bit larger number of operators. However, the operators establish through the proper links shall never be more than 5. This is because we link the numbers less than 5 to the base or subbase on the lower side and the bigger numbers are linked with the next base. For example in the previous illustrations (i) the number 19 is first linked with the nearest sub-base 20(which is further linked with the base 10 by 20=2xlo . (ii) the number 89 is linked with the nearest sub-base of 90(89 = 90-1). The number 90 is further linked with the nearest base 100, which is on the higher side. 90 = 100-10 and therefore we get 89=100-10-1. (iii) Similarly number 34 is linked with the nearest sub-base 30 which is farther linked with the base 10. Therefore 34 = 30+4 = 3x(10)+4(1)

Illustration 14: 123 = 100 + 20 + 3

= 1x(100)+2x(10)+3x(1)

which gives the operators as 1,2,3 for the hundredth, tenth and unit place.

Illustration 15 : 379 = 380 - 1 = 400 -20-1

= 4x(100)-2x(10) -1x(1)

As a matter of fact the deviation of 79 from the next sub-base can directly written by using Nikhilam Sutra as -21 etc.

Illustration 16: 429 = 400 + 30-1

= 4x(100)+3x(10)-1x(1)

Illustration 17: 889 = 1000 -111

= 1x(1000)-1x(100)-1x(10)-1x(1)

Once again the deviation can be directly written by invoking the Nikhilam Sutra. The only additional point is that the fourth operator for the thousands place has also come into operation.

Illustration 18: 1279 = 1000+300 -20-1

= 1x(1000)+3x(100)-2x(10)-1x(1)

Illustration 19: 1889 = 2000 -111

= 2000 - 100 - 10 - 1

= 2x(1000)-1x(100)-1x(10)-1x(1)

It is interesting to note that the table of a real odd number can be written down at almost magic speed by using the simplest set of operators namely 2,-1, -1 and -1 etc.

Discussion : We find surprising amount of simplicity in the Vedic approach to this facit of tables. It is always desirable to remember some tables but the vedic method gives the easy and superfast key to any of the tables and thereby the student is not afreaid of the tables. As he is confident that in case of any doubt he can write down the table in a few seconds time and thereby relieves him from the fear and pressure generally felt. And with a relaxed mind even the process of remembering tables becomes easy.

Further any table can be drawn, far far beyond the normal range of the mugged up tables, in a matter of seconds for use in any situation where we are required to multiply several times with the same number.

# CHAPTER 3

# Vinculum

## 3.1 INTRODUCTION

In the previous chapter we have seen how we can use the Nikhilam Sutra and reduce any of the operators for writing tables to be at the most equal to 5. This is a unique facet of Vedic Mathematics. Through this process the computations become far more easier than otherwise. As a matter of fact the unique concept of Vinculum in the Vedic mathematics is a very interesting and useful device.

We are generally not familiar with this approach, therefore before using this Vinculum approach for addition, multiplication, division and squaring etc. it is important to become familiar with it. Through this approach we can make it sure that we have almost never to operate digits larger than 5.

## 3.2 THE APPROACH

In the Vinculum process we take the complement of the digits bigger than 5 with a negative sign, and a positive carry over to the next higher order digit.

Illustration 1 : Let us take the number 9. 9=10-1, therefore 9 is 1 less than 10. This we can also write as $1\bar{1}$, where $\bar{1}$ at the unit place means 1 with a negative sign. For further operations we can use $1\bar{1}$ instead of 9. The property of the numbers is unchanged and we can add, multiply etc. in the usual manner. However, we have to keep in mind that the $\bar{1}$ is always appearing as -1.

In the reverse order, when we need to normalise,

$1\bar{1} = 10 - 1 = 9$

Illustration 2 : 29 = 30 - 1.

This can be written as $3\bar{1}$.

Illustration 3 : $37 = 40 - 3 = 4\bar{3}$

So we find that we take the complement of the digit larger than 5 and write this with a bar indicating the negative sig

and increase the digit at the next higher place.

Illustration 4 : $79 = 80 - 1 = 8\bar{1}$

further $80 = 100 - 20 = 1\bar{2}\bar{0}$

therefore $79 = 8\bar{1} = 1\bar{2}\bar{1}$

So we find 79 can be written as $8\bar{1}$ but since the digi the tenth place is also very big we can take its complement (from 10, 10-8 = 2) and increase the digit previous to this (0+1 = 1, therefore 79 becomes $1\bar{2}\bar{1}$)

It is interesting to note that if there are a few dig larger than 5 appearing together, their collective complemen can be directly written by using the Nikhilam Sutra and incr the previous digit by 1. In this case 9-7 = 2, 10-9 = 1, giv the complement as 21 and the previous digit 0 becomes 0 + 1 Hence $79 = 1\bar{2}\bar{1}$

Illustration 5 : 3789

By Nikhilam Sutra, the complement of the last three is 9-7 = 2, 9 - 8 = 1, 10-9 = 1, and the previous digit get increased by 1 (3+1 = 4). Therefore,

$3789 = 4\bar{2}\bar{1}\bar{1}$

Illustration 6 : 286923

The complement of the larger digits (869) by Nikhila is 131. Therefore

$286923 = 3\bar{1}\bar{3}\bar{1}23$

Note that the last digits 2 and 3 at the tenth and the unit stay on as before, and the digit previous to the bigger one increased by 1.

Illustration 7 : 353782

$353782 = 354\bar{2}\bar{2}2$

(The complement of 78 is 22)

Illustration 8 : 3728193

$$3728193 = 4\bar{3}3\bar{2}2\bar{1}3$$

The application of the Vinculum process should be well practised before we use it. However, its use makes the computation process quite easy and fast.

Illustration 9 : 782893

$$782893 = 1\bar{2}\bar{2}3\bar{1}\bar{1}3$$

Let us further consider a few examples of normalizing the Vinculum number. It shall be suprisingly interesting to learn that the Nikhilam sutra can be used even for the normalizing process. The only difference being that the previous digit is now reduced by 1.

Illustration 10 : $3\bar{3}4\bar{4}$

The complement of $\bar{3}$ is 7 and of $\bar{4}$ is 6, Further, each digit before the bar digit gets reduced by 1. So that 3-1 = 2 and 4-1 = 3.

Therefore $3\bar{3}4\bar{4} = 2736$

Illustration 11 : $3\overline{342}$

Since there are three digits in a group we can use the Nikhilam sutra to straight away write down the complement of this group, and we should remember to reduce the digit previous to this group by 1.

Therefore, by Nikhilam 9 - 3 = 6, 9 - 4 = 5 and 10-2 = 8, and 3 - 1 = 2.

As such $3\overline{342} = 2658$

Illustration 12 : $2\overline{32}6\overline{11}2$

Therefore,

$$2\overline{32}6\overline{11}2 = 1685892$$

**Illustration 13 :** $20\bar{3}\bar{4}0\bar{1}\bar{2}1$

$$20\bar{3}\bar{4}0\bar{1}\bar{2}1 = 19659881$$

Alternatively, we can normalize the above number in two steps

$$20\bar{3}\bar{4}0\bar{1}\bar{2}1 = 2\bar{1}66\bar{1}881$$

$$= 19659881.$$

**Illustration 14 :** $3\bar{6}2\bar{2}\bar{7}$

$$3\bar{6}2\bar{2}\bar{7} = 24173$$

Note:- We may not feel very comfortable to see the numbers bigger than 5 appearing with a bar, i.e. in the Vinculum form, as we would not be generally converting the smaller digits into their Vinculum. However, during the computations of the Vinculum numbers we may frequently end up in a situation like this.

Discussion :- We shall operate with the vinculum numbers exactly in the same manner as we do with the ordinary numbers without vinculum. In other words, we can understand that the property of these numbers is unchanged. However, we handle them alongwith their minus sign in the usual manner.

**Illustration 15 :** $40\bar{2}\bar{1}\bar{3}1$

Therefore, $40\bar{2}\bar{1}\bar{3}1 = 397871$

In two steps we can say

$$40\bar{2}\bar{1}\bar{3}1 = 4\bar{1}7871$$

$$= 397871$$

Further practise shall give us the grip on their system, which is immensely useful for simplifying the computation processes.

## CHAPTER 24

## OPERATION OF VINCULUM NUMBERS -PART I

### 24.1 ADDITION OF VINCULUM NUMBERS (PART I)

The addition of numbers in Vinculum form is similar to the ordinary addition the only difference being that each digit is operated alongwith its sign. The following examples shall explain the point.

24.1.1 Illustration 1: We want to add

$$\begin{array}{r} 2\ \bar{3}\ 4\ \bar{2} \\ +\ 4\ \bar{2}\ \bar{1}\ 3 \\ \hline \end{array}$$

Step 1: At the unit place 3 is to be added to $\bar{2}$ resulting in $3+\bar{2} = 3-2 = 1$.

Step 2: $\bar{1}$ is to be added to 4 at the tenth place therefore $4+\bar{1} = 4-1 = 3$.

Step 3: $-\bar{2}$ is to be added to $\bar{3}$ at the hundredth place giving us $\bar{2} + \bar{3} = -2-3 = -5 = \bar{5}$.

Step 4: 4 is added to 2 at the thousandth place. This gives $4+2 = 6$. Therefore we get

$$\begin{array}{r} 2\ \bar{3}\ 4\ \bar{2} \\ +\ 4\ \bar{2}\ \bar{1}\ 3 \\ \hline 6\ \bar{5}\ 3\ 1 \\ \hline \end{array} = 5531$$

24.1.2 Illustration 2:

$$\begin{array}{r} 3\ \bar{4}\ \bar{4}\ 2\ \bar{2} \\ +\ 7\ \bar{5}\ 3\ \bar{5}\ 4 \\ \hline \end{array}$$

Step 1: At unit place $4+\bar{2} = 4-2 = 2$.

Step 2: At tenth place $\bar{5}+2 = -5+2 = -3 = \bar{3}$.

Step 3: $3 + \bar{4} = 3-4 = -1 = \bar{1}$ at hundredth place.

Step 4: At thousandth place $\bar{4} + \bar{5} = \bar{9}$.

Step 5: $7 + 3 = 10$ therefore we get $+\ 1\ 0\ \bar{9}\ \bar{1}\ \bar{3}\ 2$

By Nikhilum sutra we write the complement of the Vinculum portion (0 9 1 3) and reduce the previous digit by 1(1-1=0)= 90872.

### 24.2 SUBTRACTION OF VINCULUM NUMBERS (PART I)

The subtraction of numbers in Vinculum form is similar to the ordinary subtraction, the only difference being that each digit is operated along with its sign. The following examples shall explain the point.

24.2.1 Illustration 3:

1. $4-3=1$
2. $3-4 = -1 = \bar{1}$
3. $4 - \bar{2} = 4-(-2) = 4+2 = 6$.
4. $\bar{3} - 2 = -3 - 2 = -5 = \bar{5}$.
5. $\bar{2} - \bar{3} = -2 -(-3) = -2+3 = 1$;
6. $\bar{4} - \bar{2} = -4-(-2) = -4+2 = -2 = \bar{2}$

The subtraction of thenumbers in the Vinculum form shall consist of only one of the above six cases. The subtraction of individual digits can be carried out without any borrowing process as shown above in case 2, 4,5 and 6.

24.2.2 Illustration 4:

$$\begin{array}{r} 5\ \bar{4}\ 2\ \bar{3}\ 1 \\ -\ 2\ \bar{2}\ 3\ \bar{4}\ 3 \\ \hline 3\ \bar{2}\ \bar{1}\ 1\ \bar{2} \end{array}$$

Step 1: $1-3 = -2 = \bar{2}$ at the unit place

Step 2: $\bar{3} - \bar{4} = -3-(\bar{4}) = -3+4 = 1$ at tenth place.

Step 3: $2 - 3 = -1 = \bar{1}$ at hundredth place.

Step 4: $\bar{4} - \bar{2} = -4-(-2) = -4+2 = -2 = \bar{2}$ at thousandth place.

Step 5: $5 -2 = 3$. Therefore we get the answer as $3\bar{2}\bar{1}1\bar{2}$, which in t normalised form gives 27908.

24.2.3 Illustration 5:

$$\begin{array}{r} 3\ 7\ 1\ 0\ 8\ 2 \\ -\ 1\ 5\ 6\ 4\ \bar{1}\ \bar{1} \\ \hline 2\ 2\ \bar{5}\ \bar{4}\ 9\ 3 \end{array}$$

Step 1: $2 - \bar{1} = 2-(-1) = 2+1 = 3$ at unit place

Step 2: $8-\bar{1} = 8-(-1) = 8+1 = 9$ at tenth place.

Step 3: $0 - 4 = \bar{4}$ at hundredth place, and similarly

Step 4: $1 - 6 = -5 = \bar{5}$.

Step 5: $7 - 5 = 2$.

Step 6: $3 - 1 = 2$ Therefore $22\bar{5}\bar{4}93 = 214693$.

24.2.4 Illustration 6:

$$\begin{array}{r} 4\;2\;6\;\bar{2}\;1\;\bar{4} \\ -\;2\;\bar{2}\;7\;\bar{4}\;4\;\bar{3} \\ \hline 2\;4\;\bar{1}\;2\;\bar{3}\;\bar{1} \end{array}$$

Step 1: $\bar{4} - \bar{3} = -4-(-3) = -4+3 = -1 = \bar{1}$

Step 2: $1-4 = -3 = \bar{3}$ at tenth place.

Step 3: $\bar{2} - \bar{4} = -2-(-4) = -2+4 = 2$, hundredth place.

Step 4: $6 - 7 = -1 = \bar{1}$

Step 5: $2 - \bar{2} = 2-(-2) = 2+2 = 4$

Step 6: $4 - 2 = 2$. Therefore $24\bar{1}2\bar{3}\bar{1} = 239169$ in the normalised form.

Illustration 7:

$$\begin{array}{r} 9\;4\;3\;\bar{5}\;1 \\ -\;7\;8\;\bar{4}\;\bar{2}\;0 \\ \hline 2\;\bar{4}\;7\;\bar{3}\;1 \end{array}$$

Step 1: $1 - 0 = 1$ at unit place.

Step 2: $\bar{5} - \bar{2} = -5-(-2) = -5+2 = -3 = \bar{3}$ at tenth place.

Step 3: $3-\bar{4} = 3-(-4) = 3+4 = 7$ at hundredth place.

Step 4: $4-8 = -4 = \bar{4}$.

Step 5: $9 - 7 = 2$.

Therefore $2\bar{\ }.7\bar{3}1 = 16671$ in the normalised form.

## 24.3 DISCUSSION

1. The above examples illustrate the operations of simple digit by digit addition and subtraction of Vinculum numbers.
2. The usual borrowing process is not required even when the digit to be subtracted is bigger than the other digit.
3. In all above examples the carry over was not required. This however may not be always true. In case the digits being added yield the sum larger than 9 the excess unity at the tenth place is carried to the next column through the simple shudhikaran process as explained in the chapter of addition.(refer Pushp 1).

## 24.4 MULTIPLICATION OF VINCULUM DIGITS

The product of digits in the vinculum form is done by using the simple algebraic rules.

Illustrations:

1. $3 \times 2 = 6$
2. $\bar{2} \times 4 = -2 \times 4 = -8 = \bar{8}$
3. $\bar{3} \times \bar{2} = -3\times(-2) = 6.$
4. $2 \times \bar{3} = 2\times(-3) = -6 = \bar{6}$
5. $\bar{4} \times 3 = -4 \times 3 = -12 = \bar{1}\bar{2}$
6. $4 \times \bar{5} = 4\times(-5) = -20 = \bar{2}\bar{0}$
7. $\bar{3} \times \bar{4} = -3 \times (-4) = 12.$

## 24.5 DISCUSSIONS

1. It is important to practice the Vinculum system a bit more as we are not familier with the same.

2. However, it is interesting to know that the Vedic Vinculum system though not be taught, is being extensively used in computers.

3. Many of the operations become much simpler when the Vinculum system is used for calculation, as shall be explained in the following chapters and subsequent volumes.

# CHAPTER 26

# MIXED ADDITION AND SUBTRACTION

## 26.1 INTRODUCTION

In the last chapter we have learnt the easy method of addition and subtraction of Vinculum numbers. With a little practise we can become confident about the simple operations of Vinculum numbers. Very frequently we have to handle problems involving mixed addition and subtraction of several numbers. The normal method would require a series of additions and subtraction. Alternatively, we can find the addition of all the positive as well as negative numbers separately. The final number is obtained by finding the difference of the two additions. But in the Vedic method it is possible to obtain the result in a single step. This is done by converting all negative numbers (i.e. the numbers to be subtracted) to digits with bar and then adding the numbers, adopting the approach of addition of Vinculum numbers.

## 26.2 ILLUSTRATIO 1:

$$374 - 253 + 245 - 433 + 327$$

STEP 1: We convert the negative numbers to the numbers with bar digits and then add.

$$\begin{array}{r} 374\\ \overline{253}\\ 245\\ \overline{433}\\ +\ 32\dot{7}\\ \hline \underline{260} \end{array}$$

STEP 2: We add the unit column $4+\bar{3} = 1$ ; $1+5 = 6$; $6+\bar{3} = 3$; $3+7 = 10$ we say Shudha and put a dot over 7, and we retain 0 at the unit place.

STEP 3: Before going to the tenth place column we look for the dots of Shudhikaran in the lower place column. In this case there may be dots over positive digits and/or negative (bar) digits. The same shall be carried over to the next column with a positive or negative sign respectively.

In this case we find only one dot in the unit place column over 7 therefore +1 is the carry over to the next column. $1+7=8$; $8+\bar{5}=3$; $3+4=7$; $7+\bar{3}=4$; $4+2=6$. It is interesting to observe that there are very few instances of carry ove as the positive and bar digits cancel each other.

STEP 4: There is no carry over from the tenth place. $3+\bar{2}=1$; $1+2=3$; $3+\bar{4}=\bar{1}$; $\bar{1}+3=2$ therefore 2 is the answer at the hundredth place. So the final answer of this mixed addition and subtraction is 260.

NOTE: In case any of the numbers consists of larger digits we may first convert them to smaller digits by using the Vinculum approach this shall have a double advantage of firstly reducing the size of the digits to be handled and secondly reducing the number of dots of Shudhikaran.

26.3 ILLUSTRATION 2: $275 - 334 + 499 - 289 + 534 - 327$.

STEP 1: Here we may convert both 499 and 289 to the Vinculum form

$499 = 50\bar{1}$ and $289 = 3\bar{1}\bar{1}$ therefore $-289 = \bar{3}11$ now we can add the numbers in the usual manner.

$$\begin{array}{r} 275 \\ \bar{3}\bar{3}\bar{4} \\ 50\bar{1} \\ \bar{3}11 \\ 534 \\ +\ \bar{3}\bar{2}\bar{7} \\ \hline 36\bar{2} \\ \hline \end{array}$$

STEP 2: We add the unit column $5+\bar{4}=1$; $1+\bar{1}=0$; $0+1=1$; $4+1=5$; $5+\bar{7}=\bar{2}$;

STEP 3: At tenth place $7+\bar{3}=4$; $4+0=4$; $4+1=5$; $5+3=8$; $8+\bar{2}=6$; so the answer at tenth place column is 6.

STEP 4: At the hundredth place column $2+\bar{3}=\bar{1}$; $\bar{1}+5=4$; $4+\bar{3}=1$; $5+1=6$; $6+\bar{3}=3$. So the answer is $36\bar{2}$. Therefore in normalised form = 358.

## 26.4 ILLUSTRATION 3:

$344 - 167 - 254 + 802 - 247 - 136$

STEP 1: Firstly we will convert the negative numbers to the bar digits and then add.

$$\begin{array}{r} 344 \\ \bar{1}6\bar{7} \\ \bar{2}5\bar{4} \\ 802 \\ \bar{2}\dot{\bar{4}}\dot{\bar{7}} \\ + \quad \bar{1}3\bar{6} \\ \hline 4\overline{58} \end{array}$$

STEP 2: $4+\bar{7}=\bar{3}$; $\bar{3}+\bar{4}=\bar{7}$; $\bar{7}+2=\bar{5}$; $\bar{5}+\bar{7}=\bar{1}\bar{2}$. So we say 'Shudha' and put a dot above $\bar{7}$ and retain $\bar{2}$ at the unit place. Further $\bar{2}+\bar{6}=\bar{8}$.

STEP 3: Since there is a dot of Shudhikaran in the previous column over $\bar{7}$, therefore $\bar{1}$ is carried over $\bar{1}+4+\bar{6}=\bar{3}$; $\bar{3}+\bar{5}=\bar{8}$; $\bar{8}+0=\bar{8}$; $\bar{8}+\bar{4}=\bar{1}\bar{2}$ so we say Shudha and put a dot over $\bar{4}$ and we retain $\bar{2}$. Further $\bar{2}+\bar{3}=\bar{5}$. Hence $\bar{5}$ is the final value of the tenth column.

STEP 4: Since there is a dot of Sudhikaran in the tenth column over $\bar{4}$, we have $\bar{1}$ as carry over.

$\bar{1}+3=2$; $2+\bar{1}=1$; $1+\bar{2}=\bar{1}$; $8+\bar{1}=7$; $7+\bar{2}=5$; $5+\bar{1}=4$ so our answer is $4\overline{58}=342$.

## 26.5 ILLUSTRATION 4:

$379 - 4065 + 5169 - 1378 - 879 + 3447 + 554 + 4676.$

STEP 1: Hence we may convert $-879$ and $379$ to the Vinculum form. Therefore $879 = 1\overline{121}$ or $-879 = \bar{1}121$ and $379 = 4\overline{21}$ now we can add the numbers in usual manner.

$$\begin{array}{r} 4\overline{21} \\ \bar{4}0\overline{65} \\ 5169 \\ \overline{1378} \\ \bar{1}121 \\ 34\dot{4}\dot{7} \\ 554 \\ 4676 \\ \hline 7903 \end{array}$$

STEP 2: We will add the unit column.
$\bar{1}+\bar{5}=\bar{6}$; $\bar{6}+9=3$; $3+\bar{8}=\bar{5}$; $\bar{5}+\bar{1}=\bar{4}$; $\bar{4}+7=3$; $3+4=7$; $7+6=13$. So we say 'Shudha' and put a dot over 6.

STEP 3: Since there is a dot in unit column over 6, we have 1 carry over. Therefore $1+\bar{2}+\bar{6}=\bar{7}$; $\bar{7}+6=\bar{1}$; $\bar{1}+\bar{7}=\bar{8}$; $\bar{8}+2=\bar{6}$; $\bar{6}+4=\bar{2}$; $\bar{2}+5=3$; $3+7=10$. Hence we say Shudha and put a dot over 7 and retain 0 at the tenth place column.

**STEP 4:** Since there is a dot of Sudhikaran over $\dot{7}$ we get 1 carry over Therefore $1+4 = 5$.

$5+\bar{0} = 5$; $5+1=6$; $6+\bar{3}= 3$; $3+1=4$; $4+4 =8$; $8+5=13$ so we say Shudha and put a dot over 5 and retain 3 at the hundredth place. Further, we add 3 to 6 which is $6+3 =9$.

**STEP 5:** $1+\bar{4}+5=2$; $2+\bar{1} = 1$; $1+\bar{1} = 0$; $0+3=3$; $3+4=7$.

### 26.6 ILLUSTRATION 5:

$2893 - 1477 + 1789 - 499 - 1434 \quad 999 + 4345 + 1434$

$+ 2343 - 1342$.

**STEP 1:** Firstly we may convert 2893, 1789, −499 and 999 to the Vinculum form. Therefore $2893 = 3\bar{1}\bar{1}3$, $1789 = 2\bar{2}\bar{1}\bar{1}$, $499= 50\bar{1}$ or $-499 = 50\bar{1}$ and $999 = 100\bar{1}$. Thus we can simply add the numbers in usual manner.

$$\begin{array}{r}
3\bar{1}\bar{1}3 \\
\bar{1}\bar{4}\bar{7}\bar{7} \\
2\bar{2}\bar{1}\bar{1} \\
\dot{\bar{5}}0\bar{1} \\
\bar{1}\bar{4}\dot{\bar{3}}\bar{4} \\
100\bar{1} \\
4345 \\
1434 \\
\dot{2}3\dot{4}3 \\
\bar{1}\bar{3}\bar{4}\bar{2} \\
\hline
1\bar{1}1\bar{3}1 \text{ Ans} \\
\hline
= 9071
\end{array}$$

**STEP 2:** Firstly we will add the unit column. $3+\bar{7}+\bar{1}+1+\bar{4}+\bar{1}+5+4+3+\bar{2}=1$.

**STEP 3:** $\bar{1}+\bar{7}=\bar{8}$; $\bar{8}+1=\bar{7}$; $\bar{7}+0=\bar{7}$; $\bar{7}+\bar{3}=\bar{1}\bar{0}$, so we say Shudha and put a dot over $\bar{3}$ and retain 0 at tenth place. Further, we add $0+0 = 0$; $0+4 = 4$; $4+3=7$; $7+4 = 11$. Here again we say Shudha and put a dot over 4 and retain 1 at the tenth place further we add $1+\bar{4}=\bar{3}$-

**STEP 4:** In the tenth column there are two dots on $\bar{3}$ and 4 so $\bar{1} + 1 =0$ is the net carryover. Hence we will add 0 to $\bar{1}$ which is $\bar{1}+0 = \bar{1}$; $\bar{1}+\bar{4} = \bar{5}$; $\bar{5}+\bar{2}=\bar{7}$; $\bar{7}+\bar{5}=\bar{1}\bar{2}$, hence Shudha an put a dot over $\bar{5}$ and we retain $\bar{2}$ at the hundredth column an further we add $\bar{2}$ to $\bar{4}$ which is $\bar{2}+\bar{4} = \bar{6}$; $\bar{6}+0 =\bar{6}$; $\bar{6}+3=\bar{3}$; $\bar{3}+4=$ $1+3 = 4$; $4+\bar{3} = 1$.

**STEP 5:** There is a dot over $\bar{5}$ in the previous column so we add $\bar{1}$ to 3 which gives $3+\bar{1}=2$; $2+\bar{1}=1$; $1+2 \;\; 3$; $3+\bar{1} =2$; $2+1=3$; $3+4=7$; $7+1=8$; $8+2=10$ so we say Shudha a put a dot over 2, and retain 0 at the thousandth place and add $0+\bar{1}=\bar{1}$. But there is a dot over 2 so we write 1 before $\overline{1}$.

### 26.7 ILLUSTRATION 6:

234 - 176 + 583 + 378 - 289

$$\begin{array}{rr} & 234 \\ & \overline{176} \\ & 583 \\ + & \dot{3}\dot{7}8 \\ & \overline{289} \\ \hline & 1\overline{27}0 \end{array}$$

$1\overline{27}0$ = 730 Ans.

### 26.8 ILLUSTRATION 7:

$$\begin{array}{r} 434 \\ \overline{154} \\ \overline{145} \\ \overline{8}11 \\ \dot{4}\dot{4}\dot{5} \\ \overline{24}\dot{\overline{4}} \\ \hline \overline{1343} \end{array} = -1543$$ A negative answer etc.

### 26.9 ILLUSTRATION 8:

124 - 243 + 435 - 134 - 342 + 244

$$\begin{array}{r} 124 \\ \overline{243} \\ 434 \\ \overline{134} \\ \overline{342} \\ \overline{2}44 \\ \hline \overline{32}3 \end{array} = -320 + 3 \quad - 317$$ Ans.

# CHAPTER 13

# THE GENERAL METHOD OF MULTIPLICATION

## 13.1 INTRODUCTION:

So far we have seen the use of the Vedic Sutras for the solution of specific types of problems. Now we shall learn the simple Vedic general method of multiplication. This method is based on the Urdhva-Tiryagvhyam Sutra. The simple meaning of this Sutra is vertically and crosswise. But this Sutra is very powerful and applicable for solving a large class of problems in mathematics. Further, it is interesting to point out that this brief Sutra has many fold applications. Prof. Nicholos and Prof. Williams have written a full book on applications of this Sutra (See news). The step by step process is outlined in the following sections:

## 13.2 SIMPLIFICATION:

For understanding the application of Urdhva Sutra it is important to first learn the method of simplification of different numbers of columns.

Level I: If there are only 2 digits in a single column we just multiply them vertically to get R the resultant number.

R = U1 x L1

1
: U
: L

For example

$\therefore$ R = 2x3 = 6 $\begin{array}{c}2\\ \underline{3}\end{array}$

If however the resulting number consists of more than one digit, the extra digit at tenth place is written below the next higher space.

Level 2 : If the simplification consists of two columns with four digits the resulting number is obtained by cross multiplication of the diagonally opposite digits and algebraic addition of the products.

$$\begin{array}{ccc} 2 & 1 & \\ \cdot & \cdot & U \\ \cdot & \cdot & L \end{array}$$

U is the upper row

L is the lower row and $U_1$ and $U_2$ are the digits in the upper row in the 1st and 2nd column respectively. Similarly $L_1$ and $L_2$ are the digits of the lower row in column 1 and 2 respectively (from right).

Therefore $R = U_1 \times L_2 + U_2 \times L_1$

For example

$$\begin{array}{cc} 2 & 1 \\ 3 & 2 \\ \hline \end{array}$$

Therefore $R = 1 \times 3 + 2 \times 2 = 7$

Level 3: If we are simplifying three columns with six digits the result shall be obtained by cross multiplying the outer column digits (column 1 and 3) and adding, we further add the product of the two digits of the central column(column 2)

$$\begin{array}{cccc} 3 & 2 & 1 & \\ \cdot & \cdot & \cdot & U \\ \cdot & \cdot & \cdot & L \end{array}$$

Therefore $R = U_1 x L_3 + U_3 x L_1 + U_2 x L_2$

For example

$$\begin{array}{ccc} 1 & 2 & 3 \\ 1 & 2 & 4 \\ \hline \end{array}$$

$R = 3 \times 1 + 4 \times 1 + 2 \times 2 = 11$

The extra digit (1) shall be written below the next higher place.

Level 4 : If we are simplifying four columns with 8 digits the result should be obtained as the summation of the four cross multiplications. The first two shall consist of the cross multiplication of the outer most two columns (1 and 4) and the remaining two shall be the cross multiplication of the inner two columns (nos 2 and 3).

$$\begin{array}{ccccc} 4 & 3 & 2 & 1 & \\ \cdot & \cdot & \cdot & \cdot & U \\ \cdot & \cdot & \cdot & \cdot & L \end{array}$$

$R = U_1 x L_4 + U_4 x L_1 + U_2 x L_3 + U_3 x L_2$

For example
$$\begin{array}{cccc} 1 & 3 & 2 & 4 \\ 1 & 2 & 3 & 5 \\ \hline \\ \hline \end{array}$$

Therefore $R = 4x1+1x5+2x2+3x3 = 22$

**Level 5** : Similarly for 5 columns
$$\begin{array}{cccccc} 5 & 4 & 3 & 2 & 1 & \\ . & . & . & . & . & U \\ . & . & . & . & . & L \end{array}$$

$$R = (U_1 x L_5 + U_5 x L_1) + (U_2 x L_4 + U_4 x L_2) + (U_3 x L_3) \text{ etc.}$$

## 13.3 THE PROCEDURE:

After understading the method of simplifying different number of columns we can now learn the step by step process of applying 'Urdhva Sutra' with the help of the following examples.

**Step 1:** Simplify the first column. We start with the extreme left hand column. This gives the extreme left hand digit of the answer.

**Step 2:** We include the second column and simplify the two columns to get the second digit of the answer.

**Step 3:** Similarly we go on including one more column, in each step, and simplify to get the subsequent parts of the answer till the first column has operated upon all other columns. This would give us the result of the product upto the levels equal to the number of columns in the digits being multiplied.

**Step 4:** For subsequent parts of the answer, starting from the first column we go on eliminating one column at each stage, and simplify the remaining columns, till we are left with the extreme left column, simplification of which gives us the extreme left part of the answer.

**Note:** In each of these simplifications if the result consists of more than one digit number, the surplus digit is written below the next higher level.

## 13.4 ILLUSTRATION 1:

Let us consider the multiplication of 2 by 2 digit numbers.

Multiply 13 x 21

$$\begin{array}{r} 21 \\ \times\ \underline{13} \\ 3 \end{array}$$

Step 1: We simplify the first column

$$R_1 = U_1 \times L_1$$

$$= 1 \times 3 = 3$$

$R_1 = 3$ is the extreme right part of the answer and is written below the first column.

Step 2 : We operate the first column with each of the subsequent columns. We include the second column which on simplification gives the next digit

$$R_2 = U_1 x L_2 + U_2 x L_1$$

$$= 1 \times 1 + 2 \times 3 = 7$$

$R_2 = 7$ is the result at the next level and is written at the tenth level.

With this we complete the operation of the first column

Step 3: We eliminate the first column and simplify the remaining columns (here only one column is left) therefore

$$R_3 = U_2 \times L_2$$

$$= 2 \times 1 = 2$$

$R_3 = 2$ is the resulting digit at the next level (hundred in this case). The result is written into appropriate level.

$$\begin{array}{r} 2\ 1 \\ \underline{\times\ 1\ 3} \\ 2/7/3 \end{array} = 273$$

## 13.5 ILLUSTRATION 2:

Multiply 26 by 34.

$$\begin{array}{r} 34 \\ \times\ \underline{26} \\ \underline{/6/4} \\ \underline{2\ 2\ \ } \end{array}$$

Step 1 : We simplify the first column

$$R_1 = U_1 \times L_1$$
$$= 4 \times 6 = 24$$

$R_1 = 24$ the result is consisting of two digits. 4 is written at the unit place and the excess digit (here 2) is written below the next level.

Step 2: Include the second column and simplify

$$R_2 = U_1 x L_2 + U_2 x L_1$$
$$= 4 \times 2 + 3 \times 6 = 26$$

$R_2 = 26$ therefore 6 is written at the tenth place and excess digit (here 2) is written below the next level.

Step 3 : We eliminate the first column and simplify

$$R_3 = U_2 \times L_2$$
$$= 3 \times 2 = 6$$

$R_3 = 6$ is written at the next level.

Step 4: We add the carryover digits to the digits at the respective level

$$\begin{array}{r} 34 \\ \times\ 26 \\ \hline 6/6/4 \\ \hline 2\ 2\ \ \ \\ \hline 8\ 8\ 4 \end{array} = \ 884$$

13.6 ILLUSTRATION 3;

Multiply 123 by 425.

$$\begin{array}{r} 4\ 2\ 5 \\ \times\ 1\ 2\ 3 \\ \hline 1/6/5 \\ \hline 2\ 1\ 1\ \ \ \ \end{array}$$

Step 1: Simplify the first column

$$R_1 = U_1 \times L_1$$
$$= 5 \times 3 = 15$$

$R_1 = 15$ therefore 5 is written in the unit column and excess digit 1 is written below the tenth column.

Step 2: We operate the first column with the subsequent columns, advancing column by column.

$R_2 = U_1 x L_2 + U_2 x L_1$

$= 5 \times 2 + 2 \times 3 = 16$

$R_2 = 16$ therefore 6 is written at the tenth place and excess digit is written below the hundredth column.

<u>Step 3</u>: We include the third column and simplify

$R_3 = U_1 x L_3 + U_3 x L_1 + U_2 x L_2$

$= 5 \times 1 + 4 \times 3 + 2 \times 2 = 21$

therefore 1 is written at hundredth place and the excess number 2 is written below the next level. This completes the operation of the first column on remaining columns.

<u>Step 4</u>: We eliminate the first column and simplify the remaining columns to compute the next part of the answer

$R_4 = U_2 x L_3 + U_3 x L_2$

$= 2 \times 1 + 4 \times 2 = 10$

$R_4 = 10$ therefore 0 is the entered in the thousandth place and excess digit 1 is written below the next level.

<u>Step 5</u>: For the next level we iliminate the second column and simplify the remaining columns. In the present example we are left with only the third column.

$R_5 = U_3 x L_3$

$= 4 \times 1 = 4$

$R_5 = 4$ is entered at the next level.

```
     4 2 5
     1 2 3
  _____________
  4/0/1/6/5
  _____________
  1 2 1 1
  _____________
  5 2 2 7 5    =   52275
```

13.7 ILLUSTRATION 4:

Multiply 7824 x 2638

```
       7824    U
     x 2638    L
    _________
    /0/4/8/2
    _________
  10/9/2/3/
  _________
```

Step 1: Simplify the first column

$R_1 = U_1 \times L_1$

$= 4 \times 8 = 32$

$R_1 = 32$ therefore 2 is written in the unit column and excess digit 3 is written below the tenth column.

Step 2: We operate the first column with the subsequent columns, advancing column by column

$R_2 = U_1 x L_2 + U_2 x L_1$

$= 4 \times 3 + 2 \times 8 =$

$= 12 + 16$

$= 28$

$R_2 = 28$, therefore 8 is written at the tenth place and 2 below the hundredth place.

Step 3: Now include the third column

$R_3 = U_1 x L_3 + U_3 x L_1 + U_2 x L_2$

$= 4 \times 6 + 8 \times 8 + 2 \times 3$

$= 24 + 64 + 6$

$= 94$

$R_3 = 94$; therefore, 4 is written at hundredth place and 9 below thousandth place.

Step 4: Including the fourth column

$R_4 = U_1 x L_4 + L_1 x U_4 + U_3 x L_2 + L_3 x U_2$

$= 4 \times 2 + 7 \times 8 + 8 \times 3 + 6 \times 2$

$= 8 + 56 + 24 + 12$

$= 100$

$R_4 = 100$ therefore 0 is written at thousandth place and the excess number 10 is written below the next level. This completes the operation of the first column on remaining columns.

Step 5: Eliminating the first column, we will simplify the remaining columns (from 2nd to 4th).

$R_5 = U_2 x L_4 + U_4 x L_2 + U_3 x L_3$

$= 2x2 + 7x3 + 6x8$

$= 4 + 21 + 48$

$= 73$

$R_5 = 73$ therefore 3 is written at ten thousandth place and 7 below the next level.

Step 6 :
Further, eliminating the second column and simplifying the remaining two columns -

$R_6 = U_3 x L_4 + U_4 x L_3$

$= 8x2 + 7x6$

$= 16 + 42 = 58$

$R_6 = 58$ therefore 8 is written at next level that is lakh, and 5 below the million level.

Step 7: Now simplifying the last column, (eliminating 3rd column also).

$R_7 = U_4 \times L_4$

$= 7x2$

$R_7 = 14$ written at the ten lakhs place

```
        7 8 2 4
        2 6 3 8
 ____________________
 14/3/3/0/4/8/2
 ____________________
  5/7/10/9/2/3 /
 ____________________
   2/0/6/3/9/7/1/2     Ans.
```

## 13.8 ILLUSTRATION 5:

Multiply    2427 x 8882

```
    2 4 2 7
  x 8 8 8 2
  _________
  /8/0/0/4
  _________
 10/8/6/1/
  _________
```

Step 1 : $R_1 = U_1 x L_1$

$= 7x2$

$= 14$

Step 2: $R_2 = U_1 x L_2 + L_1 x U_2$

$= 7x8 + 2x2$

$= 56 + 4 = 60$

Step 3: $R_3 = U_1 x L_3 + U_3 x L_1 + U_2 x L_2$

$= 7x8 + 4x2 + 2x8$

$= 56 + 8 + 16$

$= 80$

Step 4: $R_4 = U_4 x L_1 + U_1 x L_4 + U_3 x L_2 + L_3 x U_2$

$= 2x2 + 7x8 + 4x8 + 8x2$

$= 4 + 56 + 32 + 16$

$R_4 = 108$

Now the column elimination process is started.

Step 5: $R_5 = U_4 x L_2 + L_4 x U_2 + U_3 x L_3$

$= 2x8 + 2x8 + 4x8$

$= 16 + 16 + 32 = 64$

Step 6: $R_6 = U_3 x L_4 + U_4 x L_3$

$= 2x8 + 4x8$

$= 16 + 32 = 48$

Step 7: $R_7 = U_4 x L_4$

$= 2x8 = 16$

$$\begin{array}{l}
\qquad\qquad\quad 2\;4\;2\;7 \\
\qquad\qquad\quad 8\;8\;8\;2 \\
\hline
16/8/4/8/0/0/4 \\
\hline
\quad \overset{1}{6}\;\;0\;8\;6\;1 \\
\hline
1\;7\;5\;5\;6\;6\;1\;4 \quad = \quad 17556614
\end{array}$$

## 13.9 DISCUSSIONS

1. The 'Urdhava Sutra' gives us the key procedure for multiplying any two numbers. Once the method of simplification is properly understood and practised the remaining step by step procedure can be applied in a simple way. The whole process can be carried out through simple mental arthematic.

2. After a little practice even the excess carry over digit obtained at any level need not be written down below th next higher level but instead is directly added to the simplification value of the columns at the next level. If this approach is used the final answer can be obtained in one line.

3. In Vedic method we can also start computing the answer from the left hand side. This method shall be explaine in Pushp-2 along with the use of vinculum approach.

## CHAPTER 14

## MULTIPLICATION CHECK

### 14.1 Introduction

In addition to the simplicity, speed and accuracy provided by the Vedic Mathematics for the multiplication process, it also has a unique feather in its cap. There is an extremely simple 'Vedic method' for checking the correctness of the multiplication results, using which even a middle standard student can check his calculations in less than 1 minute time.

### 14.2 Procedure

The very simple, superfast and accurate procedure is outlined in the following steps with the help of an example.

14.2.1 Example 1: Multiply 1006 x 1112

The 'Nikhilam Sutra' is invoked (Refer chapter 7.

Therefore

$$\begin{array}{l} 1112 + 112 \\ 1006 + 006 \\ \hline 1118 \ / \ 672 \quad = \quad 1118672 \end{array}$$

Step 1: Write down the multiplicand, multiplier and the result in three rows in the usual manner and draw a big square in front of it. Draw the two diagonals, thereby dividing the inner space into 4 triangles.

Step 2: Add the digits of the top number. If the addition exceeds 9, once again add the digits of the result of addition. Repeat this process till the final number is a single digit number.

In this example 1+1+1+2 = 5 is the resulting single number. Write this number in the top triangle.

Step 3: Add the digits of the lower number i.e. the multiplier. If the addition exceeds 9, once again add the digits of the result of this addition. Repeat this process till a single digit number is obtained.

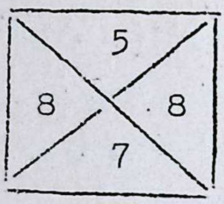
In the present example 1+0+0+6 = 7, the resulting single number has been obtained in single step.

Write this single digit number in the bottom triangle.

Step 4: Multiply the top single digit number in the top triangle with the single digit number in the bottom triangle. Write the result adjacent to the right hand side triangle. Add the digits of the resulting number. Write down this single digit number in the right hand side triangle.

1 1 1 2
1 0 0 6

1118672



$5 \times 7 = 35$
$3 + 5 = 8$

Step 5: Add the digits of the total result of the multiplication. Once again if the addition exceeds 9, add these digits of the result of previous addition. Repeat this process till the final number is a single digit number. Write this single digit number in the empty space of left hand side triangle.

Step 6: CHECK

If the number in the left hand side triangle is equal to the number in the right hand side triangle. We can be reasonably sure that the multiplication result is correct.

In the present example, we get

$1 + 1 + 1 + 8 + 6 + 7 + 2 = 26$

$2 + 6 = 8$

We observe that the numbers in the right hand side triangle and the left hand side triangle are both equal to 8. This confirms the correctness of the multiplication.

NOTE: 1. This process may be further expedited by using the following steps :

(a) As the addition of the digits is being carried out, as soon as the addition exceeds 9, we can add this sub-result digits to transform them into a single digit number. We continue the further addition, using this single digit, as the sub-result.

In the present example 1 + 1 + 1 + 8 = 11. 11 is the two digit sub-result of addition of the left hand side 4 digits.

Therefore 1+1 = 2 is the simplified single digit sub-result. This will be used for further addition.

$$2 + 6 = 8$$
$$8 + 7 = 15$$

Once again, at this stage, we get a two digit number (15) as a sub-result. This is reduced to a single digit. Therefore 1 + 5 = 6.

This sub-result is carried forward for subsequent addition. Therefore 6+ 2 = 8 is the final resulting single digit number obtained.

(b) In case there are any nines (9) in any of the numbers(the multiplicand, multiplier and the result), they can be simply eliminated during the addition process, without altering the final single digit result. This shall further simplify the already super simple process of checking. However, if any of these numbers consists entirely of nines and zeros, then only one nine should be retained, as the single digit result of the addition. Say for example the multiplier is 99090. Following this rule we can drop two nines and the resulting single digit number would be 9. Even if we add in the normal manner, we will get the same results.

9 + 9 = 18; therefore 1 + 8 = 9

9 + 0 = 0

9 + 9 = 18, therefore 1 + 8 = 9

9 + 0 = 9.

Quite obviously the 'Vedic Mathematics' procedure for checking of the multiplication is extremely simple wonderfully fast, yet quite accurate.

And at the top of it

Just 'a child's play'

## 14.3 Further Illustrations

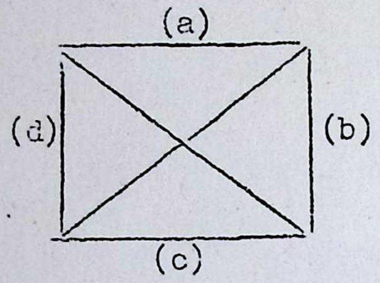
### 14.3.1 Example 2: Multiply 87965 x 99988

By applying Nikhilam we get the answer 8795444420

**Step 1:**

```
    87965
  x 99988
-----------
8795444420
-----------
```

and draw a square with four equal triangles.



**Step 2:**

Add the digits of multiplieand to get one single digit number

8 + 7 = 15, 1 + 5 = 6;

6 + 9 = 6; 6 + 6 = 12, 1 + 2 = 3; 3 + 5 = 8

Enter 8 in the upper triangle (a).

**Step 3:** Add the digits of multiplier to get one single digit number.

9 + 9 = 18, 1 + 8 =9; 9+9 = 9; 9 + 8 = 8;

We may leave all 9's and procede

8 + 8 = 16, 1 + 6 = 7;

Enter 7 in the lower triangle (c)

**Step 4:** Now multiply the digits of triangle's (ξ) and (c) i.e. 8 x 7 = 56 and convert this 56 into single digit 5+6 = 11, 1+1 = 2. Enter 2 in the R.H.S. triangle (b)

**Step 5:** Add the digits of the result to get one single digit numbe

8 + 7 = 15, 1+5 = 6; 6+ 5 = 11, 1+ 1 = 2; 2 + 4 +4 = 10,

1 + 0 = 1; 1 + 2 + 4 + 4 = 11, 1 + 1 = 2

Enter 2 in the L.H.S. triangle (d).

**Step 6:** Here 2 = 2. So our answer is correct.

14.3.2 **Example 3:** Multiply 1033 x 977

By applying Nikhilam we get the answer is 1009241. (OR) by the Urdhava sutra.

**Step 1:**

$$\begin{array}{r} 1\,0\,3\,3 \\ 9\,7\,7 \\ \hline 1009241 \\ \hline \end{array}$$

Draw a square with four equal triangle .

**Step 2:** Add the digits of multiplicand to get one single digit number

1 + 3 + 3 = 7

Enter 7 in the upper triangle(a).

**Step 3:** Add the digits of multiplier to get one single digit number.

9 + 7 + 7 = 14, 1 + 4 = 5

Enter 5 into the lower triangle(c).

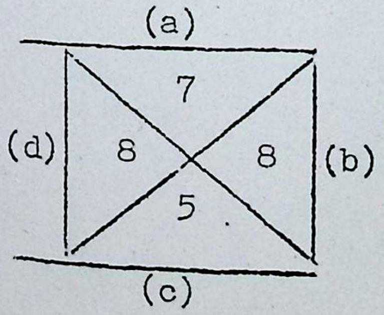
**Step 4:** Now multiply the digits of the Triange's (a) & (c) i.e. 7 x 5 = 35 and convert 35 into single digit 3 + 5 = 8. Enter 8 in the R.H.S. triangle.

**Step 5:** Add the digits of result to get one single number.

1 + 2 + 4 + 1 = 8 (Enter in the L.H.S. triangle (d).

**Step 6:** 8 = 8. So the answer is correct.

With practice the whole check can be applied mentally within a few seconds.



7 x 5 = 35;

3 + 5 = 8 .

वैदिक गणित अध्ययन

# CHAPTER 20

# COMPOUND MULTIPLICATION SQUARE AND CUBIC MEASURES

## 20.1 INTRODUCTION

By now we have seen vast applications of the simple vedic sutras, in the addition, subtraction, tables and multiplication operations. We have always found the vedic methods to be much faster than the normal methods. Very frequently we have also observed the Vedic sutras to be working at almost a magic speed. Although it is not really a magic, as it is a complete system with sound base, which can be learnt, practised and then used by any asparent. Specifically, the Vedic method for compound subtraction and compound addition is decidedly for more easier and faster. Now we come to the compound multiplication.

## 20.2 COMPUTING SQUARES

The present method provides two alternative approaches for solving problems of compound multiplication. For example, let us take the problem of finding the area of a rectangular piece of land having dimensions in yards and feet (say the plot is of size 14 yards 2 feet x 23 yeards 2 feet).

### 20.2.1 The Conventional Method

The first method is to convert both of these dimensions of the plot into the smaller unit of dimensions (here feet). Then we multiply these whole number dimensions to get the area in sq. units. Now we are required to divide this by square of the unit size to get the required area in two units.

Example: In the present case, the dimensions are 14 yards 2 feet by 23 yards 2 feet.

Therefore 14 yards 2 feet = (14 x 3) +2
= 42 + 2 = 44 feet

and 23 yards [illegible] (23 x 3) + 2
69 [illegible] 71 feet

The area is

$$\begin{array}{r} 71 \\ \times\ 44 \\ \hline 284 \\ 284\times \\ \hline 3124 \text{ sq. ft.} \end{array}$$

Now we have to convert this back into sq. yards and sq. ft.

Therefore,

$$\begin{array}{l} 9\overline{)3124}(347 \\ \ \ \underline{27} \\ \ \ \ \ 42 \\ \ \ \ \ \underline{36} \\ \ \ \ \ \ \ 64 \\ \ \ \ \ \ \ \underline{63} \\ \ \ \ \ \ \ \ \ \underline{1} \end{array}$$

Therefore, the area is 347 sq. yards and 1 sq. ft. The whole process is unnecessarily long, tedious and time consuming. Now let us try to do the same problem by using the second conventional alternative method. The first step in this case is to convert both the given dimensions into always rather vulgar fractions of the higher unit (in this case yards). This is followed by the multiplication of these fractions and then the rather tedious process of converting these fractions back to units.

Therefore, 14 yards 2 feet $= \frac{14\text{x}3}{3} + \frac{2}{3} = \frac{42}{3} + \frac{2}{3} = \frac{44}{3}$

and 23 yards 2 feet $= \frac{23\text{x}3}{3} + \frac{2}{3} = \frac{69}{3} + \frac{2}{3} = \frac{71}{3}$

Now we multiply these numbers.

$$\frac{44}{3} \text{ x } \frac{71}{3} = \frac{3124}{9}$$

Further, we have to convert this back into sq. yards and sq. feet.

Therefore,

$$\begin{array}{l} 9\overline{)3124}(347 \text{ sq. yards} \\ \ \ \underline{27} \\ \ \ \ \ 42 \\ \ \ \ \ \underline{36} \\ \ \ \ \ \ \ 64 \\ \ \ \ \ \ \ \underline{63} \\ \ \ \ \ \ \ \ \ 1 \\ \ \ \ \ \ \ \underline{\text{x}9} \\ \ \ \ \ 9)9(\ 1 \text{ sq.ft.} \\ \ \ \ \ \ \ \underline{9} \\ \ \ \ \ \ \ \underline{\text{x}} \end{array}$$

Therefore, area of the plot = 347 sq.yards and 1 sq. ft.

### 20.2.2 The Vedic Method

Both of these alternatives are too long, tedious, time-consuming and prone to error. However, the vedic approach is rather simple and straight forward. Instead of resorting to the convertion to a single unit, we use the algebraic multiplication concept and deploy the 'Adyam sutra'. It can also be done directly without the use of algebra by Urdhva Sutra(Ref.Item 20.8

Let us say that 'A' is the unit size. In the present example 1 yard = 3 ft. Therefore $A = 3$.

Side 1 = $14A + 2$ and Side 2 = $23A + 2$

We know area = (Side 1) x (Side 2)

Therefore, area = $(14A+2) \times (23A + 2)$

$= 322A^2 + 74A + 4$

Now the middle term is split into two parts. When we divide the coefficient of A by the unit size, Part 1 is the whole number quotient and Part 2 is the remainder.

Here the unit size is 3. Therefore, $74 = 72+2 = 3\times24+2$. The quotient = 24 and remainder = 2.

Therefore area = $322A^2 + (24\times3+2)A+4$.

$= 322A^2 + (24\times3+2)A+4$ (As $A = 3$) (i)

$= 322A^2 + (24A + 2)A+4$ (ii)

$= 322A^2 + 24A^2 + 2A + 4$ (iii)

$= 346A^2 + 2\times3 + 4$ (iv)

$= 346A^2 + 10 = 346A^2 + 9 + 1$ (v)

$= 346A^2 + 3\times3 + 1$ (As $A = 3$) (vi)

$= 346A^2 + A^2 + 1$ (vii)

$= 347A^2 + 1$ (viii)

= 347 sq.yards+1 sq.ft. (ix)

Since $A^2$ is the squared unit size ($3\times3=9$). So dropping $A^2$ essentially means, the coefficient is automatically expressed in the squared higher unit (here sq.yard). Further, the terms with A are transformed to the lower unit (here ft.) by

Substituting the value of A (step iv). The constant term (10) obtained at (v) step is larger than the squared unit size, we can split the same into two parts.

## 20.3 ILLUSTRATIONS

Now let us take another example. Let we take the problem of finding the area of a rectangular plot which has dimensions as 7 feet 8 inches by 5 feet 11 inches.

### 20.3.1 The Conventional Method

If we follow the first conventional method as we had done in the last example.

Therefore, 7 ft. 8 inches=(7x12)+8=84+8 = 92 inches.

and 5 ft 11 inches = (5x12)+11=60+11 =71 inches

The area is

$$\begin{array}{r} 92 \\ \underline{\times 71} \\ 92 \\ \underline{644\times} \\ \underline{6532} \text{ sq.inches} \end{array}$$

Now we have to convert this back into sq. feet and sq. inches

Therefore,

$$\begin{array}{r} 144\overline{)6532}(45 \\ \underline{576} \\ 772 \\ \underline{720} \\ \underline{52} \end{array}$$

Therefore, the area of the plot is 45 sq. feet 52 sq. inches.

Now let us try the second conventional alternative method t solve this problem.

Therefore, 7 ft.8 inches $= \frac{7x12}{12} + \frac{8}{12} = \frac{84}{12} + \frac{8}{12} = \frac{92}{12}$

and 5 ft.11 inches $= \frac{5x12}{12} + \frac{11}{12} = \frac{60}{12} + \frac{11}{12} = \frac{71}{12}$

Further, we multiply these numbers to get the area.

$$\frac{92}{12} \times \frac{71}{12} = \frac{6532}{144} = \frac{1633}{36}$$

Now we have to convert this back into sq. ft. and sq. inches.

Therefore,

```
36)1633(45 sq. ft.
   144
    193
    180
     13
   x144
36)1872(52 sq. inches
   180
     72
     72
      x
```

Therefore, the area of the plot = 45 sq.ft. and 52 sq. inches.

### 20.3.2 The Vedic Method

Let us say that A is the unit size in the present example (A=12; as 1 ft. = 12 inches). We know that the area of the plot = Side 1 x Side 2.

Therefore, Area = $(7A+8)x(5A+11)$

$= 34A^2+117A+88$

$= 35A^2+(9x12+9)A+88$

$= 35A^2+(9A+9)A+88$ (as A=12)

$= 35A^2+9A^2+9A+88$

$= 44A^2+9x12+88$

$= 44A^2+108+88$

$= 44A^2+196$

$= 44A^2+144+52$

$= 44A^2+(12x12)+52$

$= 44A^2+A^2+52$

$= 45A^2+52$

= 45 sq.ft. + 52 sq. inches

## 20.4 FURTHER ILLUSTRATIONS

We now take some more examples to clearly understand the Vedic method of computing squares. Here some examples are of finding the area of parallelograms and trapeziums to illustrate the wide use of 'Adyam' sutra in various types of cases.

### 20.4.1 Example 1

[illegible] us try to find the area of a rectangular plot measuring 3 yards 2 feet by 5 yards 1 feet. Further, let us take A as the unit size ($A=3$).

Therefore, Area $= (3A+2)x(5A+1)$

$= 15A^2+13A+2$

$= 15A^2+(4A+1)A+2$ ; (as $A = 3$)

$= 15A^2+4A^2+A+2$

$= 19A^2+4A^2+A+2$

$= 19A^2+5$

= 19 sq yards + 5 sq ft.

Here the constant term (5) obtained in the last step is smaller than the squared unit size ($3^2=9$), we do not need to split it into two parts.

### 20.4.2 Example 2

Now we will find the area of a rectangular field having dimensions 3 feet 7 inches by 8 feet 10 inches. Here the unit size, $A=12$.

Therefore, Area $= (3A+7)x(8A+10)$

$= 24A^2+86A+70$

$= 24A^2+(7A+2)A+70$

$= 24A^2+7A^2+2A+70$

$= 31A^2+24+70$

$= 31A^2+94$

= 31 sq. ft + 94 sq. inches

In the present example too, the constant term (94) being less than the squared unit size ($12^2=144$) is not divided into two parts and is simply written as 94 sq.inches.

### 20.4.3 Example 3

In this example, we will try to find the area of a grassy rectangular field having dimensions as 3 metre 22 cm by 5 metre 10 cm. Here A is the unit size ($A=100$, as 1m=100cm

Therefore, Area $= (3A+22)\times(5A+10)$

$= 15A^2+140A+220$

$= 15A^2+A^2+40\times100+220$

$= 16A^2+4220$

$= 16$ sq. m. + 4220 sq. cm.

Here too 4220 is not divided into two parts as it is smaller than the squared unit size (10,000). Till now we have confined the vedic method to finding the area of only rectangular figures. Further we shall take the examples of finding the area of the parallelogram and trapezium. The same 'Adyam' sutra is applicable in these cases too. In the case of finding the area of trapezium, we first have to find the average of the two parallel sides and then multiplying the average with the vertical distance between these two parallel sides.

### 20.4.4 Area of a parallelogram

We are interested to compute the area of a parallelogram with two adjacent sides having dimensions of 4 yards 2 feet by 8 yards 1 feet. and the distance between the two longer parallel sides is 3 yards 1 ft.

Solution

We know that the area of a parallelogram = the product of the length of any one of the two sets of the parallel sides and the vertical distance between them. In the present example, one of the two adjacent sides is 8 yards 1 ft. and the vertical distance between this set of parallel sides is 3 yards 1 ft. Therefore, by vedic method if A is the unit size (A=3); as 1 yard = 3 ft).

Area $= (8A+1)\times(3A+1)$

$= 24A^2+11A+1$

$= 24A^2+3A^2+2\times3+1$

$= 27A^2+7$

$= 27$ sq. yards + 7 sq. feet

In the present example, as the constant term (7) is less than the squared unit size, it is taken as such in the sq.ft.unit and is not divided into two parts. Therefore, Area of the parallelogram = 27 sq. yards and 7 sq. feet.

### 20.4.5 Area of a trapezium

Now we can take the example of finding the area of a trapezium with dimensions of the two parallel sides as 3 ft 10 inches (say x) by 7 ft. 8 inches (say y) and the distance between these two parallel sides is 2 ft 5 inches (say h).

**Solution**

From simple geometry, we know that the area of a trapezium with dimensions as given = $\frac{h}{2}(x+y)$. Once again if A is the unit size (here A = 12).

$$\text{Area} = \frac{2A+5}{2}\ (3A+10)+(7A+8)$$

$$= \frac{1}{2}\ (2A+5)\ x\ (10A+18)$$

$$= (2A+5)\ x\ (\frac{10A+18}{2})$$

$$= (2A+5)\ x\ (5A+9)$$

$$= 10A^2+43A+45$$

$$= 10A^2+3A^2+7A+45$$

$$= 13A^2+129$$

= 13 sq.feet + 129 sq. inches

In the present case too, the constant term 129 is not divided into two parts because it is smaller than the squared unit size and is written as such in sq. inches uni Therefore, the area of the trapezium = 13 sq. feet and 129 sq. inches.

## 20.5 THREE UNIT COMPOUND MULTIPLICATION

We have just seen how the compound multiplication becomes a pleasure by using the vedic system and thereby almost completely eleminating the needless big multiplic divisions and operations with the horrible vulgar fractions. At this stage we may ask the question that is it possible to extend the same simple pleasure-giving process of the vedic approach to compound multiplication involving more than two units at a time. This type of problems are quite frequently encountered in our day to day life. The obvious answer is 'YES'.

The step by step process of compound multiplication becomes slightly longer. But still it is far-far easier and simpler than the present system.

The following two examples with brief notes shall suffice to introduce and elaborate the step-by-step process. However, it goes without saying that once we have sufficient practice in this system, the majority of these steps can be performed mentally thereby drastically reducing the time.

### 20.5.1 Example 1

Let us find the area of a rectangular field having dimensions 3 yards 2 feet 9 inches by 5 yards 1 feet 5 inches. Here unit sizes A and B are 3 and 12 respectively.

Therefore, Area $=(3AB+2B+9)x(5AB+B+5)$

$$\begin{array}{r}
3AB + 2B + 9 \\
x \quad 5AB + B + 5 \\
\hline
15A^2B^2+13AB^2+60AB+2B^2+19B+45
\end{array}$$

$$=15A^2B^2+4x3AB^2+AB^2+5x12AB+2B^2+12B+7B+45$$

$$= 19A^2B^2+AB^2+5AB^2+3B^2+7x12+45$$

$$= 19A^2B^2+6AB^2+3B^2+129$$

$$= 19A^2B^2+2x3AB^2+3B^2+129$$

$$= 21A^2B^2+3B^2+129$$

= 21 sq. yrd. + 3 sq. ft. + 129 sq. inches

Therefore, the three-unit compound multiplication yields 21 sq. yards 3 sq. ft. 129 sq. inches.

### 20.5.2 Example 2

Let us now try to find the area of a rectangular park having dimensions as 11 yards 1 feet 4 inches by 9 yards 2 feet 11 inches. In the present example, the unit sizes are $A=3$ and $B = 12$.

$$\begin{array}{rl} \text{Therefore, Area} = & 11\,AB + B + 4 \\ & 9\,AB + 2B + 11 \\ \hline & 99A^2B^2+31AB^2+157AB+2B^2 +19B + 44 \end{array}$$

$$=99A^2B^2+10\times3AB^2+AB^2+13\times12AB+AB+2B^2+19B+44$$

$$= 109A^2B^2+14AB^2+AB+2B^2+19B+44$$

$$= 109A^2B^2+4\times3AB^2+2AB^2+AB+2B^2+19B+44$$

$$= 113A^2B^2+2\times3B^2+3B+2B^2+19B+44$$

$$= 113A^2B^2+8B^2+22B+44$$

$$= 113A^2B^2+8B^2+B^2+10B+44$$

$$= 113A^2B^2+9B^2+10\times12+44$$

$$= 113A^2B^2+9B^2+120+44$$

$$= 113A^2B^2+9B^2+144+20$$

$$= 113A^2B^2+9B^2+B^2+20$$

$$= 113A^2B^2+10B^2+20$$

= 113 sq. yards + 10 sq. ft. + 20 sq. inches

### 20.6 DISCUSSIONS

1. The present approach of compound multiplication calls for unnecessary single-unit conversions which is followed by lot of operations with the usually terrifying vulgar fractio and the whole process is inflecting the student's mind.

2. But the vedic method on the other hand makes the whole process rather simple and easy and almost completely eliminating handling of the vulgar fractions.

3. This approach also reduces the chances of error.

4. Most of the above examples have been on F.P.S. units. The obvious question may be raised that now we are already

supposed to be working, almost exclusively in M.K.S./S.I. units. But a rational view point of the realistic picture in the day to day working shall clearly indicate that lot of practical work in the field (if not most of it) is being done in F.P.S. units. Further, we should be knowing the fact that the technical journals of Europe and U.S.A. etc. (like the ACI, ASCE, etc.) have already returned back to F.P.S. units for reasons beyond our control. Therefore, if we go to the field or abroad without any orientation and practice in the F.P.S. system, we are sure to have a real tough time.

## 20.7 COMPUTING CUBES

Till now we have only seen those problems which had only two dimensions but we can extend the same methods, both present one as well as the vedic one, to the problems relating to three dimensions also. Let us try to find the volume of a parallel piped whose dimensions are 3 feet 7 inches, 5 feet 10 inches and 7 feet 2 inches. Following the present method, we will first solve it by the first conventional method and then by the second alternative.

### 20.7.1 <u>Conventional method</u> :

Therefore, 3 feet 7 inches =(3x12)+7=36+7 = 43 inches

5 feet 10 inches=(5x12)+10=60+10 = 70 inches

and 7 feet 2 inches =(7x12)+2=84 + 2 = 86 inches

Now we multiply these numbers to get the volume.

```
Therefore,      43              3010
               x70              x 86
                00             18060
              301x            24080x
              3010            258860   cu. inches
```

Now we have convert this back into cu. ft. and cu. inches.

```
Therefore,       1728)258860(149
                      1728
                       8606
                       6912
                       16940
                       15552
                        1388
```

Therefore, the volume of the parallel-piped is 149 cu.ft. and 1388 cu. inches. Now we will solve the problem by the second conventional alternative method.

### 20.7.2 Conventional method:

Therefore, 3 feet 7 inches $= \frac{3x12}{12} + \frac{7}{12} = \frac{36}{12} + \frac{7}{12} = \frac{43}{12}$

5 ft. 10 inches $= \frac{5x12}{12} + \frac{10}{12} = \frac{60}{12} + \frac{10}{12} = \frac{70}{12}$ and

7 ft. 2 inches $= \frac{7x12}{12} + \frac{2}{12} = \frac{84}{12} + \frac{2}{12} = \frac{86}{12}$.

Now we multiply these numbers to get the volume.

$$\frac{43}{12} \times \frac{70}{12} \times \frac{86}{12} = \frac{258860}{1728} = \frac{64715}{432}$$

We have now to convert this answer back into cu.ft. and cu. inches.

Therefore,

```
432)64715(149 cu. ft.
    432
    2151
    1728
     4235
     3888
      347
    x1728
     2776
      694x
    2429xx
    347xxx
432)599616(1388 cu. inches
    432
    1676
    1296
     3801
     3456
      3456
      3456
        x
```

Therefore, the volume of the parallel-piped is 149 cu.ft. and 1388 cu. inches.

### 20.7.3 The Vedic Method :

We have just seen how difficult, long and time-consuming the two current methods are. However, the vedic method even for this case of finding the volume is quite simple and straightforward.

Step 1 : The unit size is equal to 12. We first of all compute the area in the usual vedic manner.

$$\text{Area} = (3A+7) \times (5A+10)$$
$$= 15A^2+65A+70$$

Step 2 : For volume, we further multiply this by the third dimension and then transform the middle terms (have powers of A 1 and 2) to the upper power and the remainder portion to the constant term by the usual vedic split process, using the 'Adyam' sutra.

$$\text{Volume} = \begin{array}{r} 15A^2+65A+70 \\ 7A+2 \\ \hline 105A^3+30A^2+455A^2+490A+130A+140 \end{array}$$

$$= 105A^3+485A^2+620A+140$$
$$= 105A^3+12 \times 40A^2+5A^2+12 \times 51A+8A+140$$
$$= 105A^3+40A^3+5A^2+51A^2+8 \times 12+140$$
$$= 145A^3+56A^2+96+140$$
$$= 145A^3+12 \times 4A^2+8A^2+236$$
$$= 149A^3+1152+236$$
$$= 149A^3+1388$$
$$= 149 \text{ cu.ft.} + 1388 \text{ cu. inch}$$

### 20.7.4 Further Illustration :

Let us take another example of finding the volume of a parallelopiped with dimensions at 3 yards 2 feet, 6 yards 1 ft. and 5 yards 2 feet. A problem becomes equally tedious when we try to handle it by the conventional method but the Vedic method provides the straight and simple solution and that too with a great speed.

$$\text{The Area} = (3A+2)\ (6A+1)$$
$$= 18A^2+15A+2$$

$$\text{Volume} = \begin{array}{r} 18A^2+15A+2 \\ 5A+2 \\ \hline 90A^3+111A^2+40A+4 \end{array}$$

$= 90A^3 + 37\times3A^2 + 13\times3A + A + 4$

$= 127A^3 + 13A^2 + 7$

$= 127A^3 + 4\times3A^2 + A^2 + 7$

$= 131A^3 + 9 + 7$

$= 131A^3 + 16$

= 131 cu. yards + 16 cu. ft.

Remark: The simplicity, ease and comfort provided by the vedic method even in these simple problems is quite obvious.

## 20.8 DIRECT COMPOUND MULTIPLICATION

In the previous sections we have seen the use of algebraic approach for compound multiplication. In the Vedic method, we can also do the compound multiplication directly by using the 'Urdhva Sutra'.

Illustration I- Let us say we want to find the area of a plot of size 4 yards 1 feet by 8 yards 2 feet.

$$\begin{array}{r}
4\text{ yd} \quad 1\text{ feet} \\
8\text{ yd} \quad 2\text{ feet} \\
\hline
32 \;/\; 16 \;/\; 2 \\
\hline
\end{array}$$

Step I - We use the Urdhva Sutra to obtain three parts of the answer.

(1) R.H.S. = vertical product of RHS column

$= 2 \times 1 = 2$

(2) Middle part = cross multiplication

$= 1 \times 8 + 2 \times 4 = 16$

This is entered in the middle portion.

(3) L.H.S. = vertical product of LHS column

$= 4 \times 8 = 32$

Step II- Now we split the middle part into two parts. First part consists of that number which when divided by the unit size (gives the maximum whole number quotient' (here 1 yard = 3 feet, therefore 3 is the unit size). And the second part is the remainder which is always less than the unit size.

In the present example 5 x 3 = 15 is the first part (which gives the maximum whole number quotient= 5) and 16-15=1 is the second part    32 / 16 / 2

32 / 5x3+1/2

Step III- In the first part we drop the unit size (here 3) and carry the multipler (here 5) to LHS and add it to the value already there.

Here, we carry 5, therefore 32+5 = 37 square yards.

The second part (which is the remainder) is multiplied by unit size and the product carried to RHS. This is added to the value already there.

Here 1x3 = 3 is carried to RHS and added to the previous value = 2, which is already there. Therefore 2+3 = 5 square feet is the answer.

32 / 5x3+1 /2
= 32+5/ / 1x3 +2
= 37 square yards/5 square feet

This example explains the simple Vedic method of compound multiplication without the use of algebra.

Illustration II - Let us take another example. 8 feet 11 inches by 11 feet 7 inches.

| | |
|---|---|
| 8 feet | 11 inches |
| 11 feet | 7 inches |
| 88 / | 177 / 77 |

Step I- We invoke the Urdhva Sutra

(a) RHS = 11 x 7 =77

(2) Middle part = cross multiplication
= 11 x 11 + 8 x 7 = 121+56 = 177

(3) LHS = 8x11 = 88

The values are entered as usual.

Step II - Here unit size is 12 (1 feet = 12 inches). Therefore, we split the middle portion as 177 = 12x14+9.

**Step III** - We drop unit size 12, carry the multiplier 14 to LHS and add it to the value already there. Therefore 88+14 = 102. Further the second part (9) is multiplied with the unit size, carried to RHS and added to the previous value. Here we get 9x12+77 = 108+77 = 185.

88 / 177 / 77 = 88 / 12x14+9 /77

= 88 +14/ /9x12+77

= 102/ / 185

Further, we observe that the result in the lower unit (inch) is more than the squared unit size. Therefore, we carry the excess to squared feet.

Therefore 185 = 1x144+41. We carry 1 to LHS.

The result 102/185 = 102/1x144+41 = 102+1/41 = 103 square feet and 41 square inches. This may also be written as 11 square yards + 4 square feet and 41 square inches.

## 20.9 VINCULUM APPROACH

In compound multiplication, the computation work can be further simplified by first converting the larger of the smaller unit values into the vinculum form and then simplifying.

**Illustration**- 8 feet 10 inches by 10 feet 3 inches. Here we first of all convert the larger inch dimension to Vinculum form by taking its compliment from the unit size, which is written with a bar, and increase the higher unit by 1.

Here 8 feet 10 inches, the compliment of 10 inches = 12 - 10 = 2 which is written with a bar. Further the higher unit (8 feet) is increased by 1. Therefore 8+1 = 9 feet.

$$\begin{array}{lll} 9 \text{ feet} & & \overline{2} \\ 10 & & 3 \\ \hline 90 & / \; 7 \; / & \overline{6} \\ \hline \end{array}$$

**Step I** - Using Urdhva Sutra we get the three parts of the answer.

59 79

(1) RHS = $\bar{2}$ x 3 = $\bar{6}$

(2) $\bar{2}$ x 10+9x3 = -20+27 = 7, is the middle portion

(3) 9 x 10 = 90 is the LHS.

Step II- The middle portion is less than the unit size(12). Therefore, nothing is to be carried to LHS. We multiply it by the unit size and carry it to RHS. Therefore, 7 x 12 = 84+$\bar{6}$ = 78 square inch.

Hence, we get 90 / 7 / $\bar{6}$ = 90/ /7x12+$\bar{6}$

= 90 / 78 = 90 square feet and 78 square inches

This can also be written as 10 square yards and 78 square inch.

# CHAPTER - 32

## STRAIGHT DIVISION

### 32.1 INTRODUCTION:

There is yet another method of division in vedic style. This involves a bit more of mental working but the answer comes straight away in one step. The method is evolved by the combination of two vedic sutras, namely Urdhav Tiryak and Dhvajanka (meaning on top of the flag).

32.2 Let us take some examples in order to illustrate the new Dhvajanka method.

32.2.1 ILLUSTRATION 1+ Divide 46956 by 65

65 ÷ 46956

Step 1: Out of the divisor 65, we put down first digit(ie6) in the diviser column as operator and put other digit (ie5) on top of the flag as Flag digit as shown below:

↙Flag Digit
5 ÷ 46956
(operator)6

The entire division is to be done by 6.

Step 2 As one digit has been put on the top we allot one place (at the right end of D) to the remainder portion of the answer and mark off from the other digits by vertical dots

5:4695:6: - Here last digit 6 has been marked off
6

Step 3: (i) We divide 46 (of dividend) by 6 and get 7 as the quotient and 4 as remainder, we put down 7 as the first quotient digit and prefix the remainder (4) up before the next digit (i.e. 9) of the dividend. Then our actual gross dividend

(G.D) is 49 for next step.

$$\begin{array}{l} 6^{5} \; :4695\;:6: \\ \quad\;\; 4 \\ \hline \quad\;\; 7 \end{array}$$

(ii) from 49 we subtract the product of the flag digit (i.e.5) and the first quotient digit (i.e. 7). So we get 49 -

$$\begin{array}{rr} 7\text{x}5 & = 35 \\ \hline & 14 \\ \hline \end{array}$$

Thus 14 is our net divident for further operation.

Step 4: (i) Now divide this net divided i.e. 14 by 6 we get 2 as second quotient digit and 2 as remainder which is placed before the next digit of divident (i.e.5). Now our Gross dividend is 25.

$$\begin{array}{ll} 65 & :4695:6: \\ 6 & \;\;42 \\ \hline & \;\;72 \\ \hline \end{array}$$

(ii) Again from 25 we substract the product of the flag digit (5) and the 2nd quotient digit (i.e.2). So we get 25 - (2x5) = 25 - 10 = 15. This is the next net dividend.

Step 5:- (i) On dividing 15 by 6 we get 2 as third quotient digit and 3 as remainder to be placed before 6. Now 36 is out next gross dividend

$$\begin{array}{ll} \;\;5 & :4695:6: \\ 6 & \;\;423 \\ \hline & \;\;722 \end{array}$$

(ii) Out of 36 we subtract the product of the flag digit (5) and the 3rd quotient digit (i.e.2). So we get 36 - (2x5) =36-1

= 26.

Hence 26 is the remainder.

5:4695:6:
6 423
722:26:

Therefore Q = 722

R = 26 .

32.2.2 ILLUSTRATION 2: Divide 518 by 23.

23 ÷ 518

Step 1: We write it as follows

F.D.
3 ÷ 518
(operator)2

Here 3 is the flag digit

Step2: We put dots for the remainder portion

3 : 51 :8:
2

Step 3: (i) Divide 5 by 2 we get 2 as first quotient digit ,1 as the remainder and placeit before the next digit as

3 : 51 :8:
2
1
2

So our next Gross divided is 11 , 11 ND = 11 - 3x2 = 5

Step 4: Divide net divident(i.e.5) by 2, we get 2 as the 2nd quotient digit and 1 as remainder and place it before 8 as,

3 : 51 :8:
2
11
22

Next gross dividend is 18.

(ii) From 18 subtract

(3x2)=-6
12 So 12 is our remainder

```
  3 : 51 : 8            Q = 22
 2    11
   ----------           R = 12
      22 :12
   ----------
```

The whole process of step by step division as explained in the previous two examples can be done mentally and there is absolutely no need to write down the steps. By using this Vedic method of direct division we can get the total answer in one line using the simple mental working. Let us try to understand the following illustration by mentally doing the mental working as per the previous steps.

32.2.3 ILLUSTRATION 3: Divide 5120 by 64

```
   4 : 512 : 0 :
  6
        3
     ----------
      80 : 0 :
```

The second net divident is 0 [32 - 8x4]

32.2.4 ILLUSTRATION 4: Divide 7648 by 94

```
   4 :764 :8:
  9
       43
    ----------
      81 :34:
```

32.3 In the previous examples we had seen that out of the two digits of divisor we are actually dividing by using two digits and the flag digit is being used for simple readjust of the gross dividend. Now let us take few examples of divis by the three digit numbers.

32.3.1 ILLUSTRATION 5 Divide 7142695 by 824.

Here the divisor is of 3 digit. In this case we place last two digits of divisor as flag digits and adopt a slightly different modus operadi on Urdhva Tiryak lines as follows

$$\begin{array}{l} \phantom{8}24 : 71426 : 95 : \\ 8 \end{array}$$

The initial two steps are done as already explained.

Step 1 (i) Divide 71 by 8 we get 8 and remainder is 7.

$$\begin{array}{rl} 24: & 71426 :95 \\ 8 & \phantom{7}7 \\ \hline & \phantom{7}8 \end{array}$$

So 74 is the Gross dividend.

(ii) From 74 we subtract the product of the 1st digit of flag digit (i.e. 2) and the 1st quotient digit (i.e. 8). So we get 74 - (8x2) = 74 - 16 = 58. So 58 is our net next dividend.

The subsequent steps have a slight modifications. We use the Urdhva Sutra for adjustment of our gross dividend.

Step 2:(i) Divide 58 by 8 we get 6 and 10 as remainder. So our next gross dividend is 102.

$$\begin{array}{rl} 24 : & 71426 : 95 : \\ & \phantom{7}710 \\ \hline & \phantom{7}86 \end{array}$$

(ii) From 102 we subtract,(by Urdhva tiryak rule) gross products of two flag digits (24) and two quotient digits(86)i.e.

$$\begin{array}{ll} 24 & \rightarrow 12 + 32 \\ \text{x} & \\ 86 & \qquad \rightarrow 44 \end{array}$$

We get remainder is

$$\begin{array}{r} 102 \\ -\ 44 \\ \hline 58 \end{array} \quad \text{(which is our next dividend).}$$

Step 9: (i) On dividing 58 by 8 we get 6 and 10 as remainder.

Our next gross dividend is 106.

```
 24 : 71426 : 95 :
8
      7 10 10
    ----------
    8  6  6
```

(ii) From 106 we subtract, by Urdhav Tiryak rule gross products of two flag digit (24) and the last 2 quotient digits (66) i.e.

```
24
X    → 12 + 24 → 36
66
```

and the remainder is

```
 106
- 36
 ----
  70   which is our next net dividend.
```

Step 5: (i) Dividing 70 by 8 we get 8 and 6 as remainder. Out next gross dividend is 69

```
 24 : 71426 : 9 5 :
8
    7 10 10 6
  ---------------
    8  6  6 8
```

(ii) From 69 we deduct by Urdhav tiryak rule, gross product of two flag digits(24) and last 2 quotient digit 68 i.e.

```
24
X    → 16 + 24
68        →  40
```

the remainder is

```
  69
- 40
 ----
  29   (which is our net dividend )and
        295 is therefore the gross R.
```

32.3.2 <u>ILLUSTRATION</u> 6 Divide 16385 by 128

128 ÷ 16385

In this example we observe that it is more convenient to work with 1 flag digit (8 here).

Step 1 :

```
 8 ÷ 16385
12
```

Step 2: 8 : 1638 :5:
12

We make 8 as the flag digit

Step 3:(i) 8 : 1638 :5: Divide 16 by 12
12 4
1

(ii) From 43(G.D)
- 8=(8x1)
35 → next net dividend

Step 4: Divide 35 by 12 we get 2 and remainder 11.

8 : 1638:5:
4 11
12

(ii) From 118(G.D)
- 16(2x8)
102 → Next net dividend

Step 5: Divide 102 by 12 we get 8 and remainder is 6.

8 : 1638 :5:
12 4 11 6
128

(ii) From 65 (G.D)
64 (8x8)
1 Remainder

Hence 8 : 1638 :5:
12 4 11 6
128 :1:

32.3.3 ILLUSTRATION 7 Divide 24829685 by 2365

Step 1: The divisor can be written as

65 : 2 4 8 2 9 6 : 8 5 :
23

Here 23 (operator) and 65 (Flag Digits)

The entire division is to be done by 23

Step 2:- Divide 24 by 23, we get quotient digit (i.e.1) and the remainder is (i.e.1). The remainder is placed before the next digit (i.e.8) and the quotient is placed below the line

```
          6 5 : 2 4 8 2 9 6 : 8 5 :
      2 3        1
             ______________________
             1
```

Our next gross dividend is 18.

(ii) Subtract the product of the quotient digit and the first flag digit from 18(i.e. 18 - 6 = 12) which is the net dividend at this stage.

Step 3: Divide the net dividend 12 by 23 which is smaller than the divisor. So 12 is placed before the next digit (2) as our remainder (of this step) and the quotient is 0.

```
          6 5 : 2 4 8 2 9 6 : 8 5
      2 3
                 1 122
             ______________________
                 1 0
```

So our next gross dividend is 122.

(ii) Now subtract the product of the 2 quotient digits (i.e.10) and the 2 flag digits (i.e. 65) by the Urdhava Tiryak Sutra
$\begin{array}{r}65\\ \text{x}10\end{array}$ =5(by cross product)5, from 122 we get (122 - 5 = 117) which is the next net dividend.

Step 4: Again divide the net dividend (117) by 23 we get quotient digit (i.e.4) and the remainder (25) place it before the next digit (i.e.9)

```
          6 5 : 2 4 8 2 9 6 : 8 5
      2 3
                 1 12 25
             ______________________
                 1 0 4
             ______________________
```

So our next g.d. is 259.

(ii) Again subtract the product of 65 (the flag digits) and the last two digits of quotient from 259 (i.e. 259 - (65 x 04 = 24) = 235)., the cross product $\begin{array}{c}65\\ \underline{04}\end{array}$ = 24
So 235 is our net dividend.

Step 5: (i) Further divide 235 by 23, we get 9 as quotient digit and the remainder 28 is placed before the next digit (i.e. 6).

```
      6 5 : 2 4 8 2 9 6 : 8 5
2 3
            1 12 25 28
          ------------------
           1 0 4 9
```

Therefore 286 is our next gross dividend. Su32·4

(ii) Further subtract the product of the flag digit (65) and the last two digits of the quotient (i.e. 49) from 286 we get

286 - ( 65 x 49 = 74) = 212 (net dividend)

Step 6:- (i) Divide the net dividend 212 by 23 we get quotient digit 8 and the remainder 28, place it before the next digit (i.e.8)

```
      6 5 : 2 4 8 2 9 6 : 8 5
2 3
            1 12 25 28 28
          ------------------
            1 0 4 9 8 :
```

Hence 288 is next gross dividend.

(ii) Subtract the product of flag digit (65) and the last two digits of the quotient (i.e. 98) from 288. We get (288 - (65 x 98 = 93) = 105 ) 195 which is placed before the next digit 57 . So the gross R is 1955.

Step 7:- To find the net remainder again subtract the product of the last flag digit (i.e.5) and the last quotient digit (i.e.8) from 1955. We get (1955 - (8 x 5) = 40 = 1915). So 1915 is the net remainder

```
      6 5 : 2 4 8 2 9 6 : 8 5 :
2 3
            1 12 25 28 28 195
          ----------------------
            1 0 4 9 8/1 9 1 5
```

Q = 1 0 4 9 8

R = 1 9 1 5

Even these simple steps can all be performed just with mental working along with a very little rough work and the quotient and remainder obtained in a single step. Let us practice the process by doing the mental working in the following illustrations. Further in next few examples, it is more convenient to work with a single flag digit.

### 32.3.4 ILLUSTRATION:8

Divide 26786 by 114

```
 1 1 4 ÷  2 6 7 8 6
       4 : 2 6 7 8 : 6
 1 1        4 6 12
     ----------------
         2 3 4  : 1 1 0:
```

Therefore Q = 2 3 4
R = 1 1 0

See 32.4

### 32.3.5 ILLUSTRATION:9

Divide 342945 by 136

```
      6 : 3 4 2 9 4 : 5
 1 3      8 5 3 9
     ------------------
        2 5 2 1 : 8 9:
```

Therefore Q = 2521
R 89

### 32.3.6 ILLUSTRATION:10

Divide 646275 by 154

```
      4 : 6 4 6 2 7:5
 1 5      4 15 13 11
     ------------------
          4 1 9 6: 9 1:
```

Hence
Q = 4196
R = 91

32.4 **Special Cases** : In this process of division sometimes we find that the gross remainder left over is too small the necessary adjustment product is bigger than it. This problem is easily handled by just using the next lower digit as our new quotient digit. This trial and adjustment situation is equally encountered even in the division in the present method. The following example illustrates this facet.

32.4.1 ILLUSTRATION 11: Divide 601425 by 76

$$\begin{array}{l} \quad {}^{6} : 6\;0\;1\;4\;2 : 5 : \\ 7 \qquad\quad 11\;6\;3\;5 \\ \hline \qquad\quad 7\;9\;1\;3 : 3\;7 : \end{array}$$

Here in the first division by 7 we can put down 8 as a quotient digit, but the remainder then left (4) will be too small for the subtraction expected at the next step (48). So we take 7 as the quotient digit and prefix the remainder 11 to the next dividend digit and then proceed as usual.

32.5 Decimal form: The Vedic DHAWAJKAM METHOD can be easily extended to get the result in decimal form by continuing the process of division of the remainder. We place the additional zeros after the dividend. The following examples shall illustrate the process.

32.5.1 ILLUSTRATION 12: Divide 8352 by 79 to three places of Decimals.

$$\begin{array}{l} \quad {}^{9} : 8\;3\;5 : 2.\;0\;0 \\ 7 \\ \qquad\quad 14\;10\;\;83 \\ \hline \qquad\quad 1\;0\;5 : 7\;2\;1 \\ \hline \end{array}$$

The process of solving these questions is same. We put the decimal point as soon we start the division of the remainder.

32.5.2 ILLUSTRATION 13 : Divide 711.023 by 38 to three places of decimals

$$\begin{array}{l} 3^{8} : 7\;1\;1.\;0\;2\;3 \\ \qquad\quad 49\;6\;11 \\ \hline \qquad\quad 1\;8.\;711 \end{array}$$

32.5.3 ILLUSTRATION 14 : Divide 221 by 52 to three places of decimals.

$$\begin{array}{r l} & 2 : 2\ 2\ 1.\ 0\ 0 \\ 5 & \\ & \quad\ \ 2\ 3\ 1 \\ \hline & \quad\ \ 4.\ 2\ 5\ 0 \\ \hline \end{array}$$

32.5.4 ILLUSTRATION 15 : Divide 7.5 by 54 to 5 places of decimals.

$$\begin{array}{l} 4 : 7.\ 5\ 0\ 0\ 0\ 0 \\ \quad\ \ 2\ 6\ 8\ 8\ 8 \\ \hline \quad\ .\ 138888 \\ \hline \end{array}$$

## CHAPTER 33

## DIVISION CHECK

### 33. 1. Introduction

The simple 'Vedic Mathematics' procedure of checking the correctness of the multiplication results can easily be extended to check the calculations of any division of numerical numbers.

Further, it is really interesting to observe that the extremely simple Vedic Method of checking the calculations of any numerical division can be easily extended even to the algebraic division. This process is equally applicable to the problems where remainder is left over.

### 33. 2. Procedure

It shall be interesting to explain the rather simple, super fast and accurate checking procedure, with the help of an example.

Illustration 1: Divide 46956 by 65.

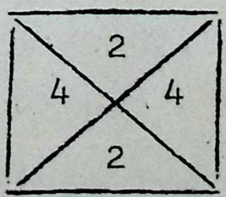
Following the straight division process, the result is obtained in two lines (refer Chapter 32. The quotient is obtained as 722 with a remainder of 26.

Step I: Subtract the remainder from the dividend (46956-26=46930 is exactly divisible by the divider 65 in this case). The result obtained is exactly divisible by the divider.

Step II: By simple observation we can conclude that if our answer of division is correct. The modified dividend obtained in Step I should be equal to the multiplication of the quotient with the divisor.

Therefore

$$\begin{array}{r} 7\ 2\ 2 \\ \times\ \ 6\ 5 \\ \hline 46930 \end{array}$$



2 x 2 = 4

Step III: The problem at this stage has become exactly the same as has been detailed out in Multiplication Check' (Chapter 14):

Therefore the exact checking procedure can be straight way applied.

(a) 7 2 2; 7 + 2 + 2 = 11; 1 + 1 = 2 (Upper triangle No.)

6 5 ; 6 + 5 = 11; 1 + 1 = 2 (Lower triangle No.)

Therefore the right hand side triangle No. is 2 x 2 = 4.

(b) 4 6 9 3 0 ; 4 + 6 = 10; 1 + 0 = 1; 1 + 9 = 10; 1 + 0 = 1

1 + 3 = 4; 4 + 0 = 4

Therefore the left hand side triangle no. is obtained as 4. The simple straight forward and superfast vedic method of checking the division has been satisfied.

33.2.1 Example 2 : Divide 876456 by 68

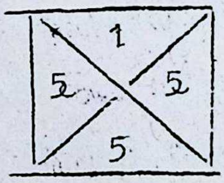
By applying the straight division process we get the quotient as 12889 with a remainder of 4.

Step 1: Subtract the remainder from the dividend

(876456 - 4 = 876452)

Step 2: The modified dividend obtained in Step I should be equ to the multiplication of the quotient with the divisor, i.e.

```
 1 2 8 8 9
 x     6 8
 ---------
   876452
```



1 x 5 = 5

Step 3: The problem at this stage has become exactly the same as has been detailed out in multiplication check.

(a) 12889 ; 1 + 2 + 8 + 8 + 9 = 28, 2 + 8 = 10, 1 + 0 = 1;

(upper triangle number) = 1

(b) 68; 6 + 8 = 14, 1 + 4 = 5

(lower triangle number

Therefore the R.H.S. triangle number is 1 x 5 = 5.

(c) 876452; 8 + 7 = 15, 1 + 5 = 6;

6 + 6 = 12, 1 + 2 = 3;

3 + 4 + 5 = 12, 1 + 2 = 3;

3 + 2 = 5.

Therefore the L.H.S. triangle number is 5.

Step 4: 5 = 5 . So our answer is correct.

33.2.2 Example 3: Divide 45696789 by 75.

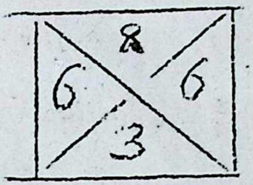
By applying the straight division process we get the quotient as 609290 with a remainder 39.

Step 1: Substract the remainder from the dividend

(45696789 - 39 = 45696750)

Step 2: The modified dividend obtained in Step 1 should be equal to the multiplication of the quotient with the divisor i.e.

```
  609290
x     75
--------
45696750
```



Step 3: The problem at this stage has become exactly the same as that of 'Multiplication check'.

(a) 609290; 6+2 = 8 (upper triangle number)

(b) 75; 7 + 5 = 12, 1 + 2 = 3 (lower triagle number)

Therefore the right hand side triangle number is

8 x 3 = 24, 2 + 4 = 6.

(c) 45696750; 4 + 5 + 6 = 15, 1 + 5 = 6; 6 + 7 = 13,

1 + 3 = 4; 4 + 5 = 9; 9 + 6 = 15, 1 + 5 = 6.

Therefore theL.H.S. triangle number is obtained as 6.

Step 4: 6 = 6. So our answer is correct.

## 33.3 Algebraic Division

Even the correctness of the division of algebraic equations can be easily checked by using the same Vedic Sutra.

The 'Vedic Mathematics' procedure outlined above for checking the division of numerical number is exactly applicable for the algebraic division as well. Only a small simple additiona step is required for transforming the algebraic problem into a numerical problem.

Step 1: We assign the value of x equal to 1. The assigned value of x (1) is substituted in all the algebraic expressions appearin in the division process and simplified. The four expressions correspond to (i) divisor, (ii) Dividend, (iii) Quotient and (iv) Remainder.

Step 2: Once the numerical values of the above four expressions have been computed, the remaining process is the exact repititio of the very simple and magic speed procedure outlined in previous section.

33.3.1 Example 4: Divide $5x^4 + 7x^3 - 3x^2 + 15x + 24$ by $x^2 - 2x + 3$

Following the steps as illustrated in previous chapter we obtain the quotient as $5x^2 + 17x + 16$ and the remainder equal to $-4x - 24$ in two lines.

Step 1: Substitute $x = 1$ and compute the numerical value.

The dividend $E = 5x^4 + 7x^3 - 3x^2 + 15x + 24$

$E = 5 + 7 - 3 + 15 + 24$

$\therefore \; D = 48$

The divisor $D = x^2 - 2x + 3$

$= 1 - 2 + 3 = 2.$

The quotient $Q = 5x^2 + 17x + 16$

$= 5 + 17 + 16 = 38.$

and the remainder $R = -4x - 24 = -4 - 24 = -28.$

Step 2: Therefore, the modified dividend is $E' = E - R$

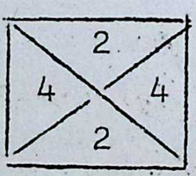
$= 48 - (-28) = 48 + 28$

$= 75$

(refer Step 1 of the previous section 2)

Therefore, the division checking problem of algebraic expressions simply reduces to a simple multiplication checking problem at this stage.

$$\begin{array}{r} 38 \\ \times\ 2 \\ \hline 76 \end{array}$$



$2 \times 2 = 4$

Step 3: Further following the procedure already explained (refer item 2). Top triangle value 38; $3 + 8 = 11$, $1 + 1 = 2$

Bottom triangle value 2.

$\therefore$ $2 \times 2 = 4$ is the R.H.S. triangle value.

L.H.S. triangle $\therefore$ 76; $7 + 6 = 13$, $1 + 3 = 4$.

The number in L.H.S. & R.H.S. triangles are equal, Hence Check of multiplication is satisfied, as much indirectly we have checked the division of even algebric expressions.

It is enteresting to note that, otherwise there is no direct, short, quick and yet certain, checking method available for the algebric division problems.

33.3.2 Example 5: Divide $9x^4+5x^3-7x^2+18x+32$ by $x^2-2x+3$.

We obtain the quotient as $9x^2+23x+12$ and the remainder equal to $-27x - 4$

CHECK

Step 1: On putting $x = 1$

The dividend $E = 9x^4 + 5x^3 - 7x^2 + 18x+32$

$= 9 + 5 - 7 + 18 + 32 = 57$

The divisor $D = x^2 - 2x + 3 = 1 - 2 + 3 = 2$

The Quotient $Q = 9x^2 + 23x + 12 = 9 + 23 + 12 = 44$

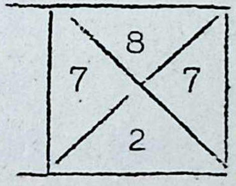
and the remainder $R = -27x - 4 = -31$

Step 2: The modified dividend as $E' = E - R$

$= 57-(-31) = 57+31 = 88$

Now it is just a multiplication checking problem.

$$\begin{array}{r} 44 \\ \times\ 2 \\ \hline 88 \\ \hline \end{array}$$



Step 3: Therefore, top triangle = 44; 4 + 4 = 8

Bottom triangle = 2

$\therefore$ 8 x 2 = 16 , the R.H.S. triangle value = 1 + 6 = 7

L.H.S. triangle $\therefore$ 88; 8 + 8 = 16, 1 + 6 = 7.

Here 7 = 7. Hence our answer is correct.

33.3.3 Example 6: Divide $3x^4 - 2x^3 - 3x - 2$ by $x^2 + 1$

We obtain the quotient $3x^2 - 2x - 3$ and the remainder is $\bar{1}x + 1$.

Step 1: On putting $x = 1$.

The dividend $E = 3x^4 - 2x^3 - 3x - 2 = 3 - 2 - 3 - 2$

$= -4 = \bar{4}$

The divisor $D = x^2 + 1 = 1 + 1 = 2$

The quotient $Q = 3x^2 - 2x - 3 = 3 - 2 - 3 = \bar{2}$

The remainder $R = \bar{1}x + 1 = \bar{1} + 1 = 0$

**Step 2:** The modified dividend is $E' = E - R$

$$= \bar{4} - 0 = \bar{4}$$

Using simple multiplication checking problem

$$\begin{array}{r} -2 \\ \times\ 2 \\ \hline -4 \end{array} \quad = \bar{4}$$

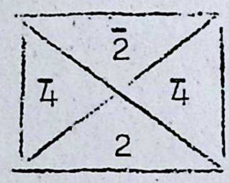
**Step 3:** Top triangle value $= \bar{2}$

Bottom triangle value $= 2$

$\therefore \bar{2} \times 2 = \bar{4}$ the R.H.S. triangle.

L.H.S. triangle $\therefore \bar{4}$

Here $\bar{4} = \bar{4}$ . Hence our answer is correct.



$\bar{2} \times 2 = \bar{4}$

## CHAPTER I

## SQUARING - URDHAVA AND YAVADUNAM SUTRA

### 1.1 URDHAVA METHOD

1.1.1 INTRODUCTION :- We have seen that how the Urdhava method is used for the multiplication of two numbers or more than two numbers. We can also use the Urdhava method for Squaring. The following examples described the procedure.

1.1.2. ILLUSTRATION :- Square the number 39

Step 1 : First we write the number as

$$\begin{array}{c} 39 \\ 39 \\ \hline \end{array}$$

Step 2 :- We draw two slash for three parts of the result.

$$\begin{array}{c} 3\ 9 \\ 3\ 9 \\ \hline R_3 / R_2 / R_1 \end{array}$$

STAGE I

Step 3 :- Multiply the digits of Ist Column, vertically and get $R_1$

$R_1 = 9 \times 9 = 81$

Step 4 :- Now multiply the digits of Ist and second column crosswise, and get $R_2$ as

$R_2 = 3 \times 9 + 3 \times 9 = 27 + 27 = 54$

OR $R_2 = 2 \times 3 \times 9 = 54$

STAGE - II

Step 5 :- Further leaving the Ist column, we multiply the digits of 2nd column only and get $R_3$.

$R_3 = 3 \times 3 = 9$

Step 6 :- Now we add the carry digits and get result.

$$\begin{array}{c} 3\ 9 \\ 3\ 9 \\ \hline 9 \,/\, 5^4 \,/\, 8^1 \end{array}$$

$= 1521$

So $39^2 = 1521$

1.1.3 ILLUSTRATION 2 :- Solve $77^2$.

Step 1 : We write as

$$\begin{array}{c} 77 \\ 77 \\ \hline \end{array}$$

Step 2 :- Draw slashes

$$\begin{array}{c} 77 \\ 77 \\ \hline R_3 \,/\, R_2 \,/\, R_1 \\ \hline \end{array}$$

STAGE I

Step 3 :- $R_1 = 7 \times 7 = 49$

Step 4 :- $R_2 = 7 \times 7 + 7 \times 7 = 49 + 49$

$= 98$

STAGE-II

Step 5:- $R_3 = 7 \times 7 = 49$

Step 6:- Add. the carry digits to get the result.

$$\begin{array}{c} 7\ 7 \\ 7\ 7 \\ \hline 49 \,/\, 98 \,/\, 49 \end{array} \quad = 49 \,/\, 9^8 \,/\, 4^9$$

$= 5929$

1.1.4. ILLUSTRATION 3 :- Solve $352^2$

The method of finding the Square of three digit number is same as described in the above two illustrations ( refer Pushp 1 Chapter 10 ).

Step 1 : We write as

3 5 2
3 5 2

Step 2 :- Draw slashes

3 5 2
3 5 2

STAGE I

Step 3:Multiply vertically the digits of Ist column , we get $R_1$

$R_1 = 2 \times 2 = 4$

Step 4 :- Now multiply the digits of $I^{st}$ & $2^{nd}$ column crosswise we get $R_2$

3 5 2
3 5 2

$R_2 = 5 \times 2 + 2 \times 5 = 10 + 10 = 20$

OR $R_2 = 2 ( 5 \times 2 ) = 20$

Step 5 :- Further multiply the digits of Ist and $3^{rd}$ column crosswise and 2nd column vertically.

$R_3 = 3 \times 2 + 3 \times 2 + 5 \times 5$

$= 6 + 6 + 25 = 37$

STAGE II Elimination

Step 6 ;- Now we leave the first column and multiply the digits of $2^{nd}$ & $3^{rd}$ column crosswise

3 5
x ,
3 5

$R_4 = 3 \times 5 + 3 \times 5 = 15 + 15 = 30$

Step 7 :- Further we multiply the digits of $3^{rd}$ Column vertically we get $R_5$ as

$R_5 = 3 \times 3 = 9$

Step 8 :- Now add the carry digits to tet the result.

$$\begin{array}{ccc} 3 & 5 & 2 \\ 3 & 5 & 2 \\ \hline \end{array}$$

$9/{}_{3}0\,/{}_{3}7/{}_{2}0/4 \quad = 123904$

1.1.5. ILLUSTRATION 4 :- Solve $772^2$

$$\begin{array}{ccc} 7 & 7 & 2 \\ 7 & 7 & 2 \\ \hline \end{array}$$

$49/\,9^{8}/7^{7}/2^{8}/4 \quad = 595984$

1.1.6. ILLUSTRATION 5:- Solve $2306^2$

$$\begin{array}{cccc} 2 & 3 & 0 & 6 \\ 2 & 3 & 0 & 6 \\ \hline \end{array}$$

$4/1\,{}^{2}/9\,/{}_{2}4/\,3\,{}^{6}/0/3^{6}$

$= 5317636$

1.2 YAVADUNAM METHOD :- There is yet another method of Squaring - This Sutra reads as follows यावदूनम् तावदूनीकृत्य वर्ग च योजयेत

Which means whatever the extent of its deficiency lesson it to the same extent and set up the square of the deficiency ( Refer Pushp 1 Chapter 9 ).

1.2.1. ILLUSTRATION 6:- Calculate the square of the number 98.

Step 1 :- First we select the base nearest to the number. Here 100 is the base.

Step 2 :- We divide the result in two parts by drawing a slash

$98^2$ = L.H.S / R. H. S.

Step 3 :- The deficiency of the number from the base is ( 100-98, = 02 ) 02.

Step 4:- Now we subtract this deficiency from the number to get the L. H. S.

as 98 - 02 = 96. So $98^2$ = 96 / R. H. S.

Step 5:- For R. H. S. result we take the square of the deficiency. So the square of 02 is 04. Hence the result is $98^2$ = 96/04

= 9604

As there are 2 zeroes in the base ( 100 ) we must have 2 digits on R. H. S.

1.2.2 ILLUSTRATION 7 : Calculate $996^2$.

L. H. S. 996 - 004 = 992

RHS = $(004)^2$ = 016

Hence $996^2$ = 992 / 016

= 992016

1.2.3. ILLUSTRATION 8 :- Calculate $1013^2$.

Base is 1000 and deviation is + 13

$1013^2$ = 1026/$(013)^2$

= 1026/169

= 1026169

Note: Here the number is bigger than the base so we add the deficiency to the number.

1.2.4. ILLUSTRATION 9 :- Calculate $100019^2$.

The base is 100000.

$100019^2$ = ( 100019 + 00019 ) / $(00019)^2$

= 100038/00361

= 100038 00361

For squaring 19 we can use the Urdhava method mentally or the same method using the base 10.

OR

( i ) 19
19

$1/_1 8/ 8^1$

= 361

(ii) $19^2$ = 19 + 09 / $(09)^2$

= $28/8^1$

base is 10 so

8 is carried to L H S.

= 28 + 8 / 1

= 36/1

= 361

1.2.5. ILLUSTRATION 10 : Calculate $1000098^2$

The base is 1 0 0 0 0 0 0

Hence

$$1000098^2 = (1000098 + 000098)/(000098)^2$$

$$= 1000196 / (000098)^2$$

But $(98)^2 = 98 - 2 / (02)^2 = 9604$

So $1000098^2 = 100196/009604$

$$= 100196^{0}09604$$

1.2.6 ILLUSTRATION 11 :- Calculate $1037^2$

The base is 1000

$$\therefore\ 1037^2 = (1037 + 037) / (037)^2$$

$$= 1074 / 0\ 3\ 7$$

$$\underline{\quad 0\ 3\ 7 \quad}$$

$$0/9/4^2/4^9 \quad = 1369$$

$$= 1074/1369$$

here R. H. S is greater than the base so we carry the excess digit 1 to the L. H. S. as

$$= 1074/_{1}369$$

$$= 1074+1/369$$

$$= 1075/369$$

$$= 1075369$$

## 1.3. Anurupena Sub-Sutra

The Application of Yavadunam can be further extended to numbers away from the base but closer to any subbase by using the Anurupena Sub-sutra. Once again only the L.H S is modified proportionately and RHS stays as it is. The deviations are taken only from the subbase ( Refer Chapter 8 Pushp 1 ).

1.3.1. Illustration 12: Square 29; Subase $= 30$

$= 3x(10)$

LHS $= 3 \ ( \ 29-1 \ ) = 3 \times 28 = 84$

RHS $= (-1)^2$ $= 1$ as base has 1 zero

$29^2 = 841$

1.3.2. ( Further Illustrations 13 :

$42^2 = 4 \ ( \ 42 + 2 \ ) \ / \ 2^2 = 4 \times 44/4 = 1764 \ ( \ 40 = 4 \times 10 \ )$

$49^2 = 5 \ ( \ 49 - 1 \ ) \ / \ 1^2 = 5 \times 48/1 = 2401 \ (50 = 5 \times 10 \ )$

$49^2 = \frac{1}{2} \ ( \ 49-1)/01^2 = \frac{1}{2} \times 48/01 = 240(1 50 = \frac{1}{2} \times 100 \ )$

$209^2 = 2 \ ( \ 209+9 \ )/ \ 09^2 = 2 \times 218/ \ 1 = 43681(200=2 \times 100 \ )$

$253^2 = \frac{1}{4} \ (253 + 3 \ )/003^2 = \frac{1}{4} \times 256/009 = 64009(250 = \frac{1}{4} \times 1000 \ )$

$507^2 = \frac{1}{2} \ ( \ 507+7) \ / \ 007^2 = \frac{1}{2} \times 514/049 = (157049 \ 500 = \frac{1}{2} \times 1000)$

$311^2 = 3(311+11)/ \ 11^2 = 3 \times 322/{}_1 21 = 9 \ 6721 \ (300 = 3 \times 100)$

The usual carry over of excess digits, prefixing of zeroes on RHS for deficit digits have to be done.

1.4: CHECKING: We can also check the squares of the numbers using the simple Vedic check method of checking the multiplication results. The following examples ( Refer chapter 11 Pushp 1 ) review the simple Procedure.

1.4.1. ILLUSTRATION 14:- Check the illustration 4 .

$772^2 = 595984$

Step 1 :- Convert the number into single digit i.

7 + 7 = 14, 1 + 4 = 5; 5 + 2 = 7 .

Step 2 :- Now Square the single digit obtained in step 1 so $7^2 = 49$, and convert the square into a single digit so 4 + 9 = 13, 1 + 3 = 4

Step 3 :- Convert the result into single digit i. e.

5 + 9 = 5, 5 + 5 = 10, 1 + 0 = 1 ;

1 + 9 = 1 ; 1 + 8 = 9; 9 + 4 = 13, 1 + 3 = 4.

CHECK : The single digit obtained in step 2 must be equal to the single digit obtained in step 3. So

4 = 4 ( Okay )

1.4.2 ILLUSTRATION 15 :- Check the results of illustration 9.

$( 100019 )^2 = 10003800361$

Step 1:- The number

i. e. 1 + 1 = 2 ; 2 + 9 = 11 ; 1 + 1 = 2

Step 2 :- Square the single digit

i. e. $2^2 = 4$

Step 3 :- Convert the result into single digit i. e. 1 + 3 = 4; 4 + 8 = 12, 1 + 2 = 3

3 + 3 = 6; 6 + 6 = 12, 1 + 2 = 3 : 3 + 1 = 4

Check so step 2 = step 3

4 = 4 ( Okay )

# CHAPTER - 2

## SQUARING - DWANDWA YOGA

2.1. INTRODUCTION:- In the previous chapter ( I ) We have learnt only the Squaring of some particular type of numbers which are near to the base or subbase, using Yavadunam Sutra. We have also learnt the urdhava method for squaring. There is another method for squaring using Dwandwa yoga sutra ( or the Duplex combination ). This sutra can be used in two ways, one is for squaring and second is the cross multiplication.

2.2. To find the Square of the numbers we find the duplexes of the digits.

2.2.1.THE PROCEDURE :- We explain the method in different levels .

Level 1 :- If we have a one digit number. To find the Duplex (D) of one digit number we simply Square the digit. For example 3, $D = 3^2 = 9$ .

Level 2 :- For two digit number. We find the duplexes of the digits as double the product of two digits, for ex.

$$35 \ , \ D = 2 ( 3x5 ) = 2 ( 15 ) = 30$$

Level 3 :- For three digit number we find the duplexes as the sum of the Square of the middle digit and double the product of the first and third digits as

$$3\ 2\ 7 \ , \ D = 2 \times 3 \times 7 + 2^2$$
$$= 42 + 4 = 46$$

Level 4 :- For four digit number we find the duplexes as the sum of, double the product of the first and fourth digits, and, double the product of the second and third digits as

$$i\ 4\ 5\ 6 = 2 \times 1 \times 6 + 2 \times 4 \times 5$$
$$= 12 + 40 = 52$$

Level 5:- We extend the same procedure for finding the duplexes of five digit number

$$1\ 2\ 3\ 4\ 6 = 2x1x6+2x2x4+3^2$$
$$= 12 + 16 + 9 = 37$$

Similarily we can extend it even for the larger digit numbers.

2.2.2. SQUARING :- The following illustration shall explain the steps in this method.

2.2.2.1. ILLUSTRATION 1 :- Calculate $53^2$.

Step 1 :- First we take the duplex of 1st digit 3, as described in level 1.

So $3^3 = 9$

i. e. the square of the first digit.

$\therefore\ 53^2 = \quad / \quad /9$

Sept 2:- Now we take the duplex of first and second digits as described in level 2, so double the product of the two digits.

$\therefore\ 53 = 2 \times 5 \times 3 = 30$

Therefore $53^2 = \ /\ {}_{3}0\ /\ 9$

Step 3 :- Further we take the duplex of 2nd digit only, as in level 1 . So square of 2nd digit is

$5 = 5^2 = 25$

$53^2 = 25/\ {}_{3}0\ /\ \ 9$

Step 4:- Add the carry digits.

$53^2 = 25\ /_{3}\ 0/\ 9$

$= 2809$

Hence $53^2 = 2809$

2.2.2.2. ILLUSTRATION 2 :- Calculate $931^2$.

Step 1 :- Duplex of Ist digit is

$1^2 = 1$

$\therefore\ 931^2 = \quad / \quad / \quad / \quad / \quad /1$

Step 2 :- Duplex of Ist and 2nd digits i. e. double the product of two digits

We get for 31 ; $2 \times 3 \times 1 = 6$

$\therefore\ 931^2 = \ / \quad / \quad /\ 6\ /\ 1$

Step 3:- Duplexes of Ist , 2nd and 3rd digits levels,we have

$931 = 2x9x1+3^2 = 18+9 = 27$

$\therefore$ $931^2$ = / / 27 / 6 / 1

Step 4:- Duplex of 2nd and 3rd digits; double the product of two digits ( level 2 ).

We have 93 = 2 x 9 x 3 = 54

$\therefore$ $931^2$ = / 54 / 27 / 6/ 1

Step 5:- Duplex of 3rd digit only ( level 1 )

We get $9^2 = 81$

$\therefore$ 931 = 81/54/27/6/1

Carry over the excess digits

= 866761

Hence $931^2 = 866761$

2.2.2.3. ILLUSTRATION 3 : Calculate $( 8342 )^2$

Step 1 :- Duplex of first digit is $2^2 = 4$

$\therefore$ $( 8 3 4 2 )^2$ = / / / / / /4

Step 2:- Duplex of Ist & 2nd digit ( level 2 )

So 42 = 2 x 4 x 2 = 16

$\therefore$ $( 8342 )^2$ = / / / / / /$1^6$/ 4

Step 3:- Duplex of Ist, 2nd & 3rd digits ( level 3 ).

$3\ 4\ 2 = 2 \times 3 \times 2 + 4^2$

= 12 + 16 = 28

$\therefore$ $( 8342 )^2$ = / / / / /$2^8$/ 16 / 4

Step 4:- Duplex of the first four digits. ( level 4 ).

8 3 4 2 = 2 x 8 x 2 + 2 x 3 x 4
= 32 + 24
= 56

$\therefore$ $( 8342 )^2$ = / / / $5^6$/ $2^8$/ $1^6$/ 4

Now we start the illumination process.

Step 5:- Duplex of $2^{nd}$, $3^{rd}$ & $4^{th}$ digits ( level 3 ) leaving Ist digit.

$834 = 2 \times 8 \times 4 + 3^2$

$= 64 + 9 = 73$

$\therefore\ (8342)^2 = /\ \ /\ 7^3/\ 5^6/\ 2^8/1^6/4$

Step 6:- Duplex of 3rd and 4th digits

i. e. $83 = 2 \times 8 \times 3 = 48$

$\therefore\ (8342)^2 = 4^8/7^3/5^6/2^8/1^6/4$

Step 7:- Duplex of 4th digit only i. e.

$8^2 = 64$

Step 8 :- Add the carry digits we get Square of the number

$(8342)^2 = 64/4^8/7^3/5^6/2^8/1^6/4$

$= 69588964$

Hence $(8342)^2 = \underline{69588964}$.

2.2.2.4. ILLUSTRATION 4:- Calculate $256745^2$ Increasing the digits in each step

Step 1 :- 5 ; $D = 5^2 = 25$

Step 2 :- 45 ; $D = 2 \times 4 \times 5 = 40$

Step 3: = 745 ; $D = 2 \times 7 \times 5 + 4^2 = 70 + 16 = 86$

Step 4 :- 6745 ; $D = 2 \times 6 \times 5 + 2 \times 7 \times 4$

$= 60 + 56 = 116$

Step 5 :- 56745 ; $D = 2 \times 5 \times 5 + 2 \times 6 \times 4 + 7^2$

$= 50 + 48 + 49$

$= 147$

Step 6 : 256745 ; $D = 2 \times 2 \times 5 + 2 \times 5 \times 4 + 2 \times 6 \times 7$

$= 20 + 40 + 84 = 144$

Eliminating the digits 144 each step.

Step 7 : 25674 ; D = 2 x 2 x 4 + 2 x 5 x 7+ $6^2$

= 16 + 70 + 36 = 122

Step 8 : 2567 ; D = 2 x 2 x 7 + 2 x 5 x 6

= 28 + 60 = 88

Step 9:- 256 ; D = 2 x 2 x 6 + $5^2$

= 24 + 25 = 49

Step 10 : 25 ; D = 2 x 2 x 5 = 20

Step 11 :- 2D; = $2^2$ = 4

Step 12 :- Now add th carry digits we get

$256745^2 = 4/{}_{2}{}^{0}/{}_{4}{}^{9}/{}_{8}{}^{8}/{}_{12}{}^{2}/{}_{14}{}^{4}/{}_{14}{}^{7}/{}_{11}{}^{6}/{}_{8}{}^{6}/{}_{4}{}^{0}/{}_{2}{}^{5}.$

= 65917995025

Hence $( 256745)^2$ = 65917995025

NOTE : There is absolutely no need to write these steps, and after a little practice the whole process is carriedout mentally.

2.2.2.5. ILLUSTRATION 5:- Calculate $( 4567423 )^2$. All this process , we can also do mentally in one line as

$( 4567423 )^2$= $4^2$/2x4x5 / 2x4x6 / 2x4x7
+ $5^2$ + 2x5x6 /

2x4x4 / 2x4x2 / 2x4x3 / 2x5x3 / 2x6x3 / 2x7x3/ 2x4x3 / 2x2x3/$3^2$
+ 2x5x7/+ 2x5x4/+2x5x2 /+2x6x2 / +2x7x2/+2x4 x 2/ + $2^2$
+ $6^2$ / + 2x6x7/ +2x6x4/ + 2x7x4
/ +$7^2$

= 16 / 40/48+25/ 56 / 32 / 16 / 24 / 30 / 36 / 42 / 24 /12/9
+60/ + 70 / + 40/ +20 + 24 +28 +16 + 4
/ + 36 / + 84 + 48 + 56 +16
+ 49

These two steps are optional

$= 16/{}_{4}{}^{0}/{}_{7}{}^{3}/{}_{11}{}^{6}/{}_{13}{}^{8}/{}_{14}{}^{0}/{}_{14}{}^{1}/{}_{11}{}^{0}/{}_{8}{}^{0}/{}_{5}{}^{8}/{}_{2}{}^{8}/{}_{1}{}^{2}/9$

= 20861352860929

Hence $( 4567423 )^2$ = 20861352860929

2.2.2.6. ILLUSTRATION 6:- Calculate $( 342564 )^2$.

We can also square the number from L.H.S. to R. H.S. in the same manner. So.

L. H. S. to R. H. S.

$$( 342564 )^2 = 3^2 / 2\times3\times4 \; , \begin{array}{c} 2\times3\times2 \\ +4^2 \end{array} / \begin{array}{c} 2\times3\times5 \\ +2\times4\times2 \end{array} / \begin{array}{c} 2\times3\times6 \\ +2\times4\times5 \\ +2^2 \end{array}$$

$$/ \begin{array}{c} 2\times3\times4 \\ +2\times4\times6 \\ +2\times2\times5 \end{array} / \begin{array}{c} 2\times4\times4 \\ +2\times2\times6 \\ +5^2 \end{array} / \begin{array}{c} 2\times2\times4 \\ +2\times5\times4 \end{array} / 2\times5\times4 + 6^2 / 2\times6\times4$$

$$/ 4^2$$

$$= 9/24/12+16 / \begin{array}{c} 30 \\ +16 \end{array} / \begin{array}{c} 36 \\ +40 \\ +4 \end{array} / \begin{array}{c} 24 \\ +48 \\ +20 \end{array} / \begin{array}{c} 32 \\ +24 \\ +25 \end{array} / \begin{array}{c} 16 \\ +60 \end{array} / \begin{array}{c} 40 \\ +36 \end{array} / 48 / 16$$

$$= 9/2^4/2^8/4^6/8^0/9^2/8^1/7^6/7^6/4^8/1^6$$

$$= 117350094096$$

DISCUSSIONS :- The Vedic methods emphasis the mental working. The detailed steps autlined in the introductory illustrations need not be written down after a little practice. The speed of computations increases with practise.

The operation of bigger digits can be eliminated by converting to vinculam form.

2.3. CHECKING :- If we want to check the results than we may apply the sutra used for checking the multiplication results.

2.3.1. ILLUSTRATION :- Check the illustration No. 3.

$$( 8342 )^2 = 69588964$$

Step 1 :- Convert the number 8342 into single digit i. e.

8 + 3 = 11, 1 + 1 = 2; 2 + 4 = 6; 6 + 2 = 8

Step 2:- Square the single digit number obtained in step 1 . i. e. $8^2 = 64$, and convert into the single digit . So

6 + 4 = 10, 1 + 0 = 1

Step 3 :- Now convert the result into single digit So.

6 + 9 = 15, 1 + 5 = 6 ; 6 + 5 = 11, 1 + 1 = 2; 2 + 8 = 10,

1 + 0 = 1; 1 + 8 = 9; 9 + 9 = 9; 9 + 6 = 6; 6 + 4 = 10, 1 + 0 = 1

CHECK : The single digit obtained in Step 2 and Step 3 are equal so the result is free from random error

Hence 1 = 1 ( Okay )

2.3.2. ILLUSTRATION 8:- . Check the illustration No. 5 :

$( 4567423 )^2 = 20861352860929$

Step 1 :-

4 + 5 = 9; 9 + 6 = 6; 6+ 7 = 13, 1 + 3 = 4;

4 + 4 = 8 ; 8 + 2 = 10 ; 1 + 0 = 1 ; 1 + 3 = 4.

Step 2 :- Square the above digit.

i. e. $4^2 = 16$; 1 + 6 = 7

Step 3 :- The result :

2 + 8 = 10, 1 + 0 = 1 ; 1 + 6 = 7; 7 + 1 = 8;

8 + 3 = 11, 1 + 1 = 2; 2 + 5 = 7 ; 7 + 2 = 9;

9 + 8 = 8; 8 + 6 = 14, 1 + 4 = 5; 5 + 2 = 7

Drop 9 .

CHECK :

7 = 7 ( Okay )

Step 2 = Step 3

The result is correct

2.4 VINCULUM FORM : If we have bigger digits, converting to vinculum form shall reduce the computational effort.

2.4.1 Illustration 9: $2199^2 = 220\bar{1}^2$

$= 4 / 8 / 4 / \dot{\bar{4}} / \bar{4} / 0 / 1$

$= \underline{\underline{4835601}}$

2.4.2 Illustration 10: $3298^2 = 330\bar{2}$

$$= 9 \,/\, {}_{1}8 \,/\, 9 \,/\, {}_{\bar{1}}\bar{2} \,/\, {}_{\bar{1}\,1}\bar{2} \,/\, 0 \,/\, 4$$

$$= 1088\bar{3}\,\bar{2}04 = \underline{\underline{10876804}}$$

2.4.3. Illustration 11 : $20289^2 = 203\bar{1}\bar{1}^2$

$$= 4 \,/\, 0 \,/_{1} 2 \,/\, \bar{4} \,/\, 5 \,/\, \bar{6} \,/\, \bar{5} \,/\, 2 \,/\, 1 = 4\,1\,2\,\bar{4}\,5\,\bar{6}\,\bar{5}\,2\,1$$

$$= 411643521$$

2.5 Discussion : With this we came to the end of another very interesting chapter of Vedic Methods for the Squaring of any number and checking of the solution obtained.

One again we observe the emphasis on metal workin This approach when regularly practised leads to a more agile a alert mind.

## CHAPTER - 3

## SQUARE ROOTING

3.1 INTRODUCTION: To find the Square root of the number using the Vedic Sutra is very simple, easy and interesting. The sub sutra named Dwandwa Yoga( द्वन्द योग ) or Duplex process has two meaning. First one is by squaring and the second one is cross multiplication. This sutra is very easy to apply. Using this sutra we can also find the squares of the numbers. In chapter 9 of Pushp-I we have seen a different method of squaring. But that method is applicable only for particular type of numbers.

3.2. THE PROCEDURE: The use of the above Sutra is explained below:

Computing Duplexes :

Level 1 :- If we have a single digit number than we take Square of number.

example = 4, than Duplex $4^2 = 16$

Level 2:- We have a two digit number then twice of the product of the two digits of the number.

For example for 23 Duplex = 2 x 2 x 3 = 12

Level 3 : For three digit number, the sum of double of the product of the first and third digit of the number, and , square of the middle digit is the Duplex

example for 223 Duplex $=2\text{x}2\text{x}3 + 2^2$

$= 12 + 4 = 16$

Level 4:- For four digit number, the sum of the double of the product of the first and forth digit and double of the products of second and third digit.

for 2426 duplex = 2 x 2 x 6 + 2 x 4 x 2

= 24 + 16 = 40

Level 5:- For five digit number, the sum of the double of the product of the first and fifth digit, double of the product of the second and fourth digit and the square of the third digit gives the duplex.

For 24652 Duplex $= 2 \text{ x } 2 \text{ x } 2 + 2 \text{ x } 4 \text{ x } 5 + 6^2$

= 8 + 40 + 36 = 84

The same method is used for the larger numbers.

3 .2.1. SQUARE ROOT:- Using the above method the step by step process of computing the Square root is illustrated in the following examples.

..1. ILLUSTRATION 1 : Find the Square root of 4489.

Step 1 :- First we make the pairs of two digits each from the given number R. H. S. to L. H. S. and write as

44 : 89 :

The first pair of two digits from L. H. S. is written outside the dots and rest of the pairs are written in between the dots.

Step 2:- Now we find the square of the first nine natural numbers as 1 , 4, 9, 16, 25, 36, 49, 64, 81 and choose one of them which is most suitable and lesser than the first pair

(i) In this case square of 6 is suitable. So 6 is the first quotient digit. After this step we find the remaining part of the square root using straight division process.

So subtract the square of 6 ( i. e. 36 ) from 44 we get ( 44 - 36 = 8 ) 8 as a remainder which is prefixed as the first digit of the next pair ( i. e. 8 ). Hence 88 is the dividend.

(ii) For finding the divisor we double the first quotient digit ( i. e. 2 x 6 = 12 ) obtained in step 2 (i) and write as follows

$$\begin{array}{r l}
 & 44 : {}_{8}89 : \\
12 & \\
\hline
 & : 6 : \\
\hline
\end{array}$$

So 12 is our divisor. The entire division is done by 12.

Step 3:- Now divide 88 by 12 we get 7 as a quotient and 4 as the remainder which is prefixed to the next digit 9. So 49 is our next gross dividend

$$\begin{array}{r l}
 & 44 : {}_{8}8{}_{4}9 : \\
12: & \\
\hline
 & : 6 : 7 : \\
\end{array}$$

Now subtract the square of the quotient digit ( i. e. $7^2=49$) obtained in ( Step 3 (1) ) for the gross dividend 49 we get ( 49 - 49 = 0 ) 0 as quotient and 0 as remainder. Hence the square root is 67. which is an exact square.

DISCUSSION : (1) We have just seen how easy it is to find the square root of any big number.

(2) If a number contains n digits

(i) If n is odd than the square root of a number shall have $\frac{n+1}{2}$ number of digits.

(ii) If n is even than the square root of number shall have $\frac{n}{2}$ digits.

(iii) If the number is of <u>pure decimal</u> then the number of digits in the square is double that in the square root.

(iv) If the square root is not an exact number, the decimal shall be put after the number of digits equal to the number of pairs formed.

3.2.2.2. ILLUSTRATION 2 :- Find the Square root of 567009.

Step 1 :- First we make the pairs of the number and place in between the dots as shown.

56 : 70 09 :

Step 2 :- Now we find the suitable digit of first pair, Here 49 is most suitable lesser digit. So 7 is the first quotient digit, Subtract 49 from 56 we get 7 as a remainder, which is prefixed to the next digit (7) so 77 is the dividend.

The divisorn is double of the first quotient digit 7; we get 14. So 14 is the divisor and we write as

```
        56 : 70 09 :
             :7
     14:
    ----------------------
         : 7 :
```

Step 3:- Divide 77 by 14 we get 5 as a quotient digit and 7 as a remainder, which is perefixed to the next digit 0 . So 70 is the next gross dividend.

56 : 7 0 0 9 :
14: :7 7 :
: 7 : 5 :

ii) Subtract the square of the quotient digit 5 ( i.e. $5^2 = 25$) ( level 1 ) from the gross dividend ( 70 ) to get the ne dividend. ( 70 - 25 = 45 ; 45 is the net dividend ).

Step 4 :- Now divide 45 by 14 we get 3 as the next quotient digit and 3, the remainder which is placed before the next digit 0. So 30 is next gross dividend.

56 : 7 0 0 9 :
14: :7 7 3 :
: 7 : 53 :

Subtract the double of the product of two quotient digits 5 & 3 ( Using Dwan dwa yoga Sutra ( level 2)from 30, we get , the net dividend. 2 x 5 x 3 - 30 = 30-30=0 0. So 0 is net dividend .

Step 5:- Now 0 is not divisible by 14 So 0 is both the quotient and remainder which is placed before the next digit 9 .

Now 09 is the gross dividend.

56 : 7 0 0 9 :
14 : :7 7 3 0 :
:7 : 5 30 0 :

(ii) Subtract the sum of the double of the product of the quotient digits 5 and 0 and the square of the middle digit 3 ( i. e. $2 \times 5 \times 0 + 3^2 = 9$ ) from 9(level 3 ); we get ( 9 - 9 = 0 ; 0 as quotient and 0 as remainder.

Hence the square root of 567009 is 753. This is an exact square root case.

### 3. 2. 3. ILLUSTRATION 3 : Find the square root of 17 .

```
17
    17 : 0   0   0   0  0   0   0   0
   8:   :1  2   3   2  2   6   7   3
  ___________________________________
 : 4    :1  2   3   1  0   5   6
```

the net dividend at each stage are

10

$20 - 1^2 = 20 - 1 = 19$

$30 - 2 \times ( 1 \times 2 ) = 30 - 4 = 26$

$20 - ( 2 \times (3 \times 1 ) + 2^2 ) = 20 - 10 = 10$

$20 - ( 2 \times ( 1 \times 1 ) + 2 \times ( 2 \times 3 ) = 20 - 14 = 6$

and so on. Hence the square root of $\sqrt{17}$ is 4.1231056 which is not an exact square root .

3.2. ILLUSTRATION 4 : Find the square root of 229441

Step 1 :- First we make the pairs of two digits each of the number from R. H. S. to L. H. S. and write as.

22 : 94 41 :

The first pair from L. H. S. is written before the first dot and other pairs are written in between the dots.

Step 2 :- (i) Now we find the square of the first nine natural numbers as 1, 4, 9, 16, 25, 36, 49, 64, 81. In this case we take 4 as a first digit of the square root because 16 is last smaller number than 22 and the remainder is 6 which is prefixed to the next number ( here 9 ). After this step we find the remaining part of the square root, using straight division process.

So 69 is our next dividend.

(ii) For finding the divisor We double the first digit of the square.root

22 : 9 4 4 1 :
8: 6

: 4 :

(i) Here double of 4 = 8

So 8 is our divisor and we write as

22 : 94 4 1 :
8: 6

: 4 : :

Step 3: We divide 69 by 8 ~~we~~ use 7 instead of 8 ( If we take 8 as the quotient digit remainder is 5, giving gross = 54; After using Dwandwa yoga Sutra for getting net dividend, the Square of the quotient digit ( level 1 ) is greater than the gross dividend,

8 x 8 = 6 4 7 5 4 ) as the second quotient digit and 13 as a remainder which is prefixed before 4. So 134 is our next gross dividend.

8 : 22 : 9441 :
:6 13 :

:4 : 7 :

(ii) Subtract the square of the quotient digit ( $7^2 = 49$ ) 49 from the gross dividend 134, we get ( 134 - 49 = 85 ) 85 as the net dividend.

Step 4: Now 85 is divided by 8 we get 9 instead of 10 as a third quotient digit and 13 as a remainder, prefixed before the next digit 4. So 134 is the next gross dividend.

22 : 9 4 4 1 :
8: 6 13 13 :

: 4 : 79 :

(ii) Now subtract the double of the product of the quotient digits ( level 2 ) as ( 2 x 7 x 9 = 1 2 6 ) 126 from the gross dividend 134, We get ( 134 - 126 = 8 ) 8 is the next net dividend.

Step 5: Again we divide 8 by 8 we use 0 instead of 1 ( see step 3 ) So 0 is the next quotient digit and 8 is the remainder

22 : 9 4 4 1
8: 6 13 13 8

: 4 : 7 9 . 0 :

81 is the next gross dividend.

(ii) Again we subtract the double of the three quotient digits ( ∵ 2 x 7 x 0 + $9^2$ = 81 ) ( level 3 ) from 81(gross dividend ). We get ( 81 - 81 = 0 ). 0 as both/the quotient and remainder. Hence our work has been completed

∴ So the square root of 229441 is 479 which is a perfect number.

### 3.2.5. ILLUSTRATION 5 : Find the square root of 16641.

1 : 6 6 4 1 ;
2 : : 0 2 4 8

: 1 : 2 9 . 00 : ( Complete )

The result is 129.

Notes;(i) After making the two digit pairs of the number from R. H. S. to L. H. S. if a single digit is left then it is counted as a pair ( of single digit ).

(i) When we divide 6 by 2 after step 2, we write 2 as a quotient digit instead of 3, if we write 3 as Q we get remainder 0 and the next gross dividend is 06 which is smaller than the square of the Q digit 3 ( i. e 6 9 ) which would give the negative dividend value. So we take 2 as the quotient digit

(iii) Due to the same reason we do not divide the net dividend 8 by 2 ( obtained in steps 5 (ii).

Vinculum Approach : We can also proceed by using vinculum numbers.

1 : 66 4 1 :
0 $\bar{1}$ 0
2

1 : 3 $\bar{1}$ 0 0

Square root = $13\bar{1}$ = 129 ; is the exact square root

Net dividend

$1^2 = 1$

2 x 1 = 2 Divisor

06 - 3 x 3 = $\bar{3}$

$\bar{1}$ 4 - 2 x 3 x $\bar{1}$ = - 6 + 6 = 0.

01 - ( 2 x 3 x 0 + $\bar{1}$ x $\bar{1}$ ) = 1 - 1 = 0

3.2. ILLUSTRATION 6 : Find the square root of 1145678

1 : 1 4 5 6 7 8
: 0 2: : 0 1 0 5 1 5

:1: 0 7 0 . 36 ( in complete

The net dividend at each stage is

0 1
14 - 0 = 14
5 - 0 = 5
56 -72 = 7
17 - 0 = 0

and soon

Hence the square root of 1145678 is not a perfect number.

So $\sqrt{1145678}$ = 1070.36 ---------

3.3. Square - root of decimal numbers : The Vedic method for finding the square root of decimal number is perfectly the same.

3.3.1. ILLUSTRATION 7 : Find the square root of 547.56

: 5 : 47 . 56
4 : 12 21
: 2 : 3 . 4 0 0 :

The single digit 5 on LHS is counted as a pair by itself. The net dividend at each stage is.

1 4
2 7 - $3^2$ = 1 8
2 5 - 3x4x2 = 1
1 6 - $4^2$ = 0

Hence the square of the number is 23.4 -

Notes -1. As there are 2 pairs on LHS of the decimal point we have 2 digits on L H S of decimal point The remaining digits shall be after the decimal point ( on R H S ).

3.3.2. ILLUSTRATION 8 : Find the square root of the number .000015.

First we make the pairs of the number as .00 00 15 gives 15 as the first non zero digit pair. The square root of the two pair of zero is .00 according to the discussion. So we find only the square root of 15.

Hence

: 1 5 : 0 0 0 0
6 :- 6 12 14 10
: 3 : 8 7 3 0 --

Hence the square root is .0038730 of the given number.

3. 3.3. ILLUSTRATION 9: Find the square root of 1 2 4 3 . 5 9

: 1 2 : 4 3 . 5 9 0 0
6 : 3 4 / 6 9 11 16
: 3 : 5.26 4 7 --------

Hence the square root is 35.2647 etc.

## 3.4. VINCULUM APPROACH :

3.4.1. ILLUSTRATION 10:- Let us try to solve the Illustration 8 again by using vinculum approach.

$. 00\ 00\ 15$ for $15$

$15;\ 0\ \ 0\ \ 0\ \ 0\ \ 0$

$6\ \ 6\ \ \bar{3}\ \ 6\ \ \bar{3}$

$6$

$3.\ 9\ \bar{3}\ 3\ 0\ 2$

$= 3.8730$ etc.

Hence . 0038730 is the square root.

3.4.2. Illustration 11: Solve again illustration 9 using vinculum.

$:12:4\ 3\ .\ 5\ 9\ 0\ 0\ 0$

$34\ \ 0\ \ \bar{1}\ 2\ 0\ 0$

$6$

$3:5\ 3\ \bar{4}\ 5\ \bar{1}\ \bar{6}$

$35.2648$ etc.

3.5 Discussion: 1 In this chapter we have studied the straight, simple and easy Vedic method of finding the square root of any number.

2. The Dawadva Yoga Sutra used for squaring of numbers has been employed again even for computing the square root.

3. The step by step Vedic method of Square rooting supersedes the present methods in many respects.

4. The checking method can be easily and directly applied for checking the correctness of results in case of the exact square roots.

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## CHAPTER 8

## EVALUATION OF DETERMINANTS

Reference: Vertically and Crosswise

### 8.1 Sum of Products

First Method Here we employ the sutra 'by alternate elimination and retention.'

8.1.1 Illustration 1 : Calculate 6 x 9 + 7 x 8 + 4 x 3

Each of the three products is a two-digit figure; eliminating the first figure in each case, we have 4 + 6 + 2 (the last figures of 54, 56 and 12). Hence the last digit is 2, with 1 to carry. Now we eliminate the last digits, giving: 1 (carried) + 5 + 5 + 1 = 12

Answer 122

8.1.2 Illustration 2: Calculate 7 x 9 + 6 x 4 + 3 x 8

Last digits give : 3 + 4 + 0 = 7

First digits give :6 + 2 + 4 = 12

Answer 127

8.1.3 Illustration 3: 6x 7 + 18 x 9 + 14 x 6 + 3 x 8

Here we observe that the four products in question are : 42, $2x9^2$ = 2 x 81 = 162, 2 x 7 x 6 = 2 x 42 = 84 and 24

Last digits give: 2+2+4+4 = 2 and 1 to carry

First digits give: 1 (carried) +4 + 16 + 8 + 2 = 31

Answer 312

Note that for each product the last digit is given by the product of the last digits. But here in the last example we are beginning to stray into the territory of the second method, which will now be considered.

8.2. Second Method This draws on the 'vertically and crosswise' sutra

### 8.2.1 Illustration 4: Calculate 43 x 72 + 54 x 31

We are going to carry out two 'vertically and crosswise' multiplications simultaneously. In the units place we have:

3 x 2 + 4 x 1 = 10

Write down 0, and carry 1

So far we have:

$$\begin{array}{ccccc} 43 & & & & 54 \\ \times & & + & \times & \\ 72 & & & & 31 \\ \hline & & & & 0 \\ & & & 1 & \end{array}$$

Next, in the tens place we have:

1 + 4 x 2 + 3 x 7 + 5 x 1 + 4 x 3 = 47

Note the scope here for employing the first method. We now write down 7, and carry 4. Thus, so far we have written:

$$\begin{array}{cccc} 4 & 3 & 5 & 4 \\ \underline{7} & \underline{2} & \underline{3} & \underline{1} \\ & & 7 & 0 \\ & 4 & 1 & \end{array}$$

Next, for the hundreds place, we have:

4 (carried) + 4 x 7 + 5 x 3 = 47

Hence the answer is 4770.

A more convenient layout for the calculation is shown in the following example.

### 8.2.2 Illustration 5 Evaluate 213 x 426

+ 354 x 631

+ 652 x 172

$$\begin{array}{cccc} \times & 2 & 1 & 3 \\ & 4 & 2 & 6 \\ + & & & \\ \times & 3 & 5 & 4 \\ & 6 & 3 & 1 \\ + & & & \\ \times & 6 & 5 & 2 \\ & 1 & 7 & 2 \\ \hline \end{array}$$

$$4\;2\;{}_{10}6\;{}_{12}2\;\;{}_{5}5\;\;{}_{2}6$$

The larger digits, such as the 7's, and 8's and 9's, can be avoided by using the vinculum.

## 8.3 Second Order Determinants

Methods for mental evaluation of second order determinants and more generally of sums of products, make use of the preceding methods.

## 8.4 Third Order Determinants

A third order determinant is a layout, or matrix of elements specifying nine multiplications and various additions and subtractions, leading finally to a single figure. Thus a third order determinant is a formula reducing $3^2 = 9$ figures to one.

8.4.1 Illustration 6 : Evaluate $D = \begin{vmatrix} 2 & 6 & 7 \\ 5 & 4 & 8 \\ 3 & 1 & 9 \end{vmatrix}$

Let us separate the first row from the second and third by a partition:

$$D = \left|\begin{array}{ccc} 2 & 6 & 7 \\ \hdashline 5 & 4 & 8 \\ 3 & 1 & 9 \end{array}\right|$$

Three second-order determinants arise from the second and third rows :

$$\begin{vmatrix} 5 & 4 \\ 3 & 1 \end{vmatrix} = 7 \; ; \; \begin{vmatrix} 5 & 8 \\ 3 & 9 \end{vmatrix} = 2 \; ; \; \begin{vmatrix} 4 & 8 \\ 1 & 9 \end{vmatrix} = 28$$

The sign of the second of these is reversed (by calculating 8 x 3-5x9 = -21), and the three results, 7, -21, and 28, are noted in a fourth row:

$$\begin{array}{ccc} 2 & 6 & 7 \\ \hdashline 5 & 4 & 8 \\ 3 & 1 & 9 \\ \hline -7 & -21 & 28 \end{array}$$

The first and fourth rows are now multiplied together 'vertically and cross-wise', i.e. we take

$$2 \times 28 + 6 \times (-21) + 7 \times (-7) = -119$$

$$\therefore \quad D = -119$$

8.4.2 Illustration 7: Evaluate $E = \begin{vmatrix} -3 & 7 & 1 \\ 2 & 4 & 3 \\ 5 & 8 & -4 \end{vmatrix}$

$$\begin{array}{cc} & \begin{array}{ccc} -26 & -11 & +17 \end{array} \\ \therefore \; E = & \left|\begin{array}{ccc} -3 & 7 & 1 \\ 2 & 4 & 3 \\ \hdashline 5 & 8 & -4 \end{array}\right| \end{array}$$

$$\underline{E = +277}$$

8.4.3 Illustration 8 : $F = \begin{array}{r} 37 \\ -32 \\ 19 \end{array} \left|\begin{array}{cc:c} 7 & -3 & 9 \\ 3 & 4 & 5 \\ -1 & 5 & 7 \end{array}\right|$

$$= \underline{270}$$

Note that the partitioning has been applied in various places; in Example 3 it is by columns. The multiplication pattern,

now becomes,

Since interchanging rows and columns does not change a determinant, evidently the sutra, 'vertically and cross-wise', needs to be understood as applying to both these patterns.

Note also that, of the three second-order determinants calculated after partitioning, that with negative sign contains elements spaced one row (or column) apart; adjacent elements lead to a positive sign. This observation is useful in dealing with fourth and higher order determinants. The following example should make the point clear.

## 8.5 Fourth Order Determinants

Partitioning centrally (either horizontally or vertically) we now proceed to evaluate second order determinants either side of the partition.

### 8.5.1 Illustration 9:

Evaluate $D = \begin{vmatrix} 2 & 5 & 7 & 8 \\ 3 & 6 & 2 & 4 \\ 3 & 5 & 7 & 6 \\ 1 & 6 & 9 & 7 \end{vmatrix}$

$$\begin{array}{cccccc} + & - & + & + & - & + \\ -3 & -17 & -16 & -32 & -28 & 12 \end{array}$$

$$= \left|\begin{array}{cccc} 2 & 5 & 7 & 8 \\ 3 & 6 & 2 & 4 \\ \hline 3 & 5 & 7 & 6 \\ 1 & 6 & 9 & 7 \end{array}\right|$$

$$\begin{array}{cccccc} 13 & 20 & 15 & 3 & -1 & -5 \end{array}$$

The second order determinants above the partition are:

$\begin{vmatrix} 2 & 5 \\ 3 & 6 \end{vmatrix} = -3$ , $\begin{vmatrix} 2 & 7 \\ 3 & 2 \end{vmatrix} = -17$ ; $\begin{vmatrix} 2 & 8 \\ 3 & 4 \end{vmatrix} = -16$

$\begin{vmatrix} 5 & 7 \\ 6 & 2 \end{vmatrix} = -32$ ; $\begin{vmatrix} 5 & 8 \\ 6 & 4 \end{vmatrix} = -28$ ; $\begin{vmatrix} 7 & 8 \\ 2 & 4 \end{vmatrix} = 12$

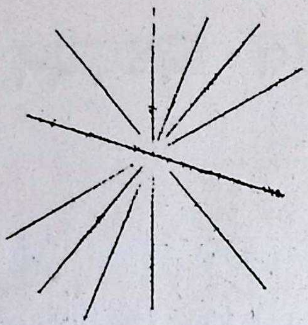
These are placed above the determinant in a row, the associated signs being placed above, '+' for adjacent columns '-' for those spaced one column apart, '+' for those spaced two apart.

The second-order determinants below the partition are obtained in the same sequence:

$\begin{vmatrix} 3 & 5 \\ 1 & 6 \end{vmatrix} = 13$ ; $\begin{vmatrix} 3 & 7 \\ 1 & 9 \end{vmatrix} = 20$ ; $\begin{vmatrix} 3 & 6 \\ 1 & 7 \end{vmatrix} = 15$

$\begin{vmatrix} 5 & 7 \\ 6 & 9 \end{vmatrix} = 3$ ; $\begin{vmatrix} 5 & 6 \\ 6 & 7 \end{vmatrix} = -1$ ; $\begin{vmatrix} 7 & 6 \\ 9 & 7 \end{vmatrix} = -5$

The two outermost rows of figures are now multiplied together according to the pattern,



associating the relevant signs with them. i.e. we have:

$$D = +(-3)(-5)-(-17)(-1)+(-16)(3)+ \ldots\ldots + (12)(13)$$

$$= \underline{186}$$

**EXAMPLES**

1. $D = \begin{vmatrix} 3 & 2 & 1 \\ 2 & 7 & 1 \\ 5 & 6 & 5 \end{vmatrix}$ (D = 54)

2. $D = \begin{vmatrix} 4 & 1 & 8 \\ 6 & 2 & 3 \\ 2 & 1 & 0 \end{vmatrix}$ (D = 10)

3. $D = \begin{vmatrix} 2 & 3 & 2 \\ 4 & 7 & 3 \\ 2 & 1 & 3 \end{vmatrix}$ (D = -2)

4. $D = \begin{vmatrix} 4 & 5 & 5 & 7 \\ 7 & 6 & 8 & 8 \\ 2 & 2 & 2 & 3 \\ 3 & 4 & 9 & 4 \end{vmatrix}$ (D = 9)

5. $D = \begin{vmatrix} 2 & 9 & 4 & 3 \\ 5 & 6 & 7 & 1 \\ 2 & 4 & 3 & 5 \\ 1 & 4 & 3 & 2 \end{vmatrix}$ (D = 145)

6. $D = \begin{vmatrix} 3 & 2 & 4 & 6 \\ 2 & 5 & 7 & 1 \\ 2 & 4 & 1 & 3 \\ 5 & 4 & 6 & 2 \end{vmatrix}$ (D = 414)

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## CHANGE OF ROOTS OF EQUATION

10.1 . INTRODUCTION: In this chapter we consider how to change a polynomial to obtain another whose roots are related to the original equation by being -

A. Opposite in sign.
B. Reciprocals.
C. Multiples or sub-multiples
r D. Increased or reduced by a given amount .

10.2 To obtain an equation whose roots are opposite in sign to those of a given equation.

We transpose the sign of every alternate term, beginning with the second:

10.2.1. Illustration 1 :-

1) $x^4-2x^3 - 13x^2 + 38x -24 = 0$ has roots 1, 2, 3, - 4.

$x^4+2x^3 - 13x^2 - 38x -24 = 0$ has roots -1, -2, - 3, 4.

10.2.2. Illustration 2 :-

2) $2x^2 + 5x - 3 = 0$ has roots $\frac{1}{2}$ , - 3.

$2x^2 - 5x - 3 = 0$ has roots $-\frac{1}{2}$, 3.

10.2.3. Proof If $f(x) = 0$ is a polynomial equation, and if $-y = x$, then $f(-y) = 0$ is satisfied by every root of $f(x) = 0$ with its sign transposed . $f(-y) = 0$ has roots opposite in sign to those of $f(x) = 0$, and since successive terms in any polynomial alternate between odd and even powers of the unknown, substituting $-y$ for $x$ effectively changes the sign of alternate terms.

10.3. To obtain an equation whose roots are reciprocals of those in a given equation.

We reverse the order of the coefficients:

10.3.1 Illustration 3 :- $2x^3 + 3x^2 -8x + 3 = 0$ has roots $\frac{1}{2}$, 1, -3.

the coefficients are 2,3, -8, 3,

$3x^3 -8x^2 + 3x + 2 = 0$ has roots 2, 1, $-\frac{1}{3}$.

Illustration 4 :-

10.3.2 $15x^2 -13x + 2 = 0$ has roots $\frac{2}{3}$ , $\frac{1}{5}$ .

$2x^2 - 13x + 15 = 0$ has roots $\frac{3}{2}$ , 5.

वेदात् सर्वं प्रसिद्ध्यति
वेद सम्पूर्ण ज्ञान के अनादि अनन्त स्रोत हैं।
**Vedas are the Eternal Sources of All Knowledge**

10.3.3. Proof  If $f(x) = 0$ is a polynomial equation, and if $x = \frac{1}{y}$, then $f(\frac{1}{y}) = 0$ will have roots reciprocal to those of $f(x)=0$. But making the substitution $x=\frac{1}{y}$ will, on multiplying through by the appropriate power of y give a polynomial equation with reversed coefficients.

Eg. If $ax^2 + bx + c = 0$

$$\frac{a}{y^2} + \frac{b}{y} + c = 0 \quad \text{becomes } a + by + cy^2 = 0$$

$$\text{ie. } cy^2 + by + a = 0$$

10.4 To obtain an equation whose roots are all equal to those of a given equation multiplied by a given number.

This comes under the 'Proportionately' formula of Vedic mathematics.

We multiply the coefficients of the given equation by successive powers of the given number, starting with zero:

10.4.1 Illustration 5 : $x^2 + 2x - 15 = 0$ has roots 3, − 5.

Suppose we want an equation with double those roots, ie. 6, −10. Multiply the coefficients by $2^0$, $2^1$, $2^2$:

$$x^2 + 2.2x - 2^2.15 = 0$$

or $x^2 + 4x - 60 = 0$ is the required equation.

10.4.2. Illustration 6 : $x^2 + 2x - 15 = 0$ has roots 3, − 5.

For an equation with roots 9, −15 ie. 3 times the original roots:

$$x^2 + 6x - 135 = 0.$$

For an equation with roots $1\frac{1}{2}$, $-2\frac{1}{2}$ ie. half the original roots:

$$x^2 + x - 3\frac{3}{4} = 0$$

or $4x^2 + 4x - 15 = 0$

10.4.3. Illustration 7 : $2x^3 - 3x^2 - 2x + 3 = 0$ has roots $1\frac{1}{2}$, −1, 1.

An equation with twice these roots is

$2x^3 - 6x^2 - 8x + 24 = 0$ ie. multiply by 1, 2, 4, 8.

$x^3 - 3x^2 - 4x + 12 = 0$

10.4.4. Illustration 8 : $2x^3 - 3x^2 - 2x + 3 = 0$ as above

We can in fact start with any term and multiply by powers of 2 to the riht, and divide by powers of 2 to the left. Thus we can multiply this equation by $\frac{1}{2}$, 1, 2, 4 to double its roots:

$$x^3 - 3x^2 - 4x + 12 = 0.$$

10.4.5. Illustration : $6x^2 - x - 15 = 0$ find an equation with 3 times the roots.

Multiply by $\frac{1}{3}$, 1, 3

$$2x^2 - x - 45 = 0$$

10.4.6. Proof If $f(x) = 0$ is a polynomial equation, and if $y = xn$, then $f(\frac{y}{n}) = 0$ will have roots n times those of $f(x) = 0$.

But if $f(\frac{y}{n}) = 0$ then $y^p . f(\frac{y}{n}) = 0$ where p is the order of the polynomial.

Eg. If $ax^2 + bx + c = 0$

$$\frac{ay^2}{n^2} + \frac{by}{n} + c = 0$$

or $$ay^2 + nby + n^2c = 0.$$

# CHAPTER - 11

# THE SOLUTION OF LINEAR AND NON-LINEAR DIFFERENTIAL EQUATIONS

## 11.1. INTRODUCTION :

The method of series solution of differential equations is well-known. Bringing to bear the vedic mathematical system of sutras of Sri Bharati Krishna Tirthaji makes this a very powerful approach, as is shown by the following examples.

The procedure is applicable to a wide range of problems, in particular those where boundary conditions are given at the origin or at a single point.

We start with a simple linear example to illustrate the method. Except perhaps for the lay-out the procedure of the first example is familiar to western mathematicians.

### 11.1.1. Illustration 1.

Solve $2y + 3y' = 18 + 8x$ ............. (1)

given $y'(0) = 4$

Here, and throughout this Chapter, $y' = \frac{dy}{dx}$

and $y'(0) = \frac{dy}{dx}$ evaluated with $x = 0$

An explanation of the steps will be given shortly, but the complete working and solution appears finally as follows:-

$$y \quad \overset{3}{a} + \overset{4}{b}x + \overset{0}{c}x^2 + \overset{0}{d}x^3 + \overset{0}{e}\,x^4 + \ldots\ldots\ldots\ldots \quad (2)$$

$$y' = \underset{4}{b} + \underset{0}{2cx} + \underset{0}{3dx^2} + \underset{0}{4ex^3} + \underset{0}{5fx^4} + \ldots\ldots\ldots\ldots \quad (3)$$

i.e., the solution is, $\underline{y = 3 + 4x}$ exactly

**Explanation**

By Maclurin's Theorem, a series expansion can be used for y, as in Equation (2). Differentiating gives Equation (3), successive coefficients of x being arranged in successive columns. Purely for purposes of explanation we will now use the following layout:-

| Coefficients | | | | | Column | | |
|---|---|---|---|---|---|---|---|
| | 1 | 1 | 2 | 3 | 4 | | |
| 2 | $y$ | $= a$ | $+ bx$ | $+ cx^2$ | $+ dx^3$ | + ........ | Row 1 |
| 3 | $y'$ | $= b$ | $+ 2cx$ | $+ 3dx^2$ | $+ 4ex^3$ | + ........ | Row 2 |
| | | | 18 + 8x | | | | Row 3 |

The left-hand side of Equation (1) translates into 2x Row 1 + 3x Row 2, and the coefficients '2' and '3' have here been placed in a column on the left as a reminder. To facilitate the working, the right-hand side terms of Equation (1) can be placed in a third row. Thus for each successive column we have,

2 x Row 1 + 3 x Row 2 = Row 3,

in accordance with Equation (1), And each successive column yields the next of the unknown, a, b, c, d, ............

The sutra here is, 'by alternate elimination and retemtion'

Since $y'(0) = 4$, $b = 4$, from Row 2

Write this in place, as shown in the complete solution.

To find 'a', Column 1 ( absolute terms only ) gives us,

$$2^{A} + 3b = 18$$

Since b is known to equal 4, the mental procedure is, reduce 18 by 3 x 4, giving 6, and divide by 2 to obtain a = 3. Now write '3' above 'a' in Row 1.

So far we have:

$$y = \overset{3}{a} + \overset{4}{bx} + cx^2 + \dots\dots\dots\dots$$

$$y' = \underset{4}{b} + 2cx + 3dx^2 + \dots\dots\dots\dots$$

Similarly , in Column 2,

$$2b + 3 \times 2c = 8$$

$$2c = 8 - 2 \times 4 = 0$$

$$c = 0, \text{ etc.}$$

The same procedure shows that all subsequent terms are zero, and

$$y = 3 + 4x \text{ exactly,}$$

The next example introduces a non-linearity, namely the term $y^2$.

### 11.1.2. Illustration 2 :

Solve $y'' = y^2 + 1 + 2x^2 + x^4$ ............... (1)

Where $y(0) = 0$, and $y'(0) = 0$

The working can be written down as follows:

Column I II III IV V VI VII VIII IX

$$y = \overset{0}{a} + \overset{0}{b}x + \overset{\frac{1}{2}}{c}x^2 + \overset{0}{d}x^3 + \overset{\frac{1}{6}}{e}x^4 + \overset{0}{f}x^5 + \overset{\frac{1}{40}}{g}x^6 + \overset{0}{h}x^7 + \overset{\frac{1}{336}}{i}x^8 + \overset{0}{j}x$$

$$y'' = \overset{1}{2c} + \overset{0}{6dx} + \overset{2}{12ex^2} + \overset{0}{20fx^3} + \overset{\frac{3}{4}}{30gx^4} + \overset{0}{42hx^5} + \overset{\frac{1}{6}}{56ix^6} + \overset{0}{72jx^7} + ..$$

**Explanation**

From the boundary conditions we have a = 0 and b = 0 As before, we work with coefficients of x, in succession , each step yielding another of the values of b, c, d, ..........
To conform with the left-hand side of Equation (1), the successive terms for $y^2$ are given by successive duplexes ( or dwandwa-yogas)+ of Equation (2)

**For 'c'**

From Column 1,

$$2c + a^2 = 1,$$

Since we are eliminating all except the absolute terms. And since $a^2 = 0$, the mental procedure is, reduce 1 by 0 to obtain 1 as the valu of 2 c. Hence we put 2c = 1 and $c = \frac{1}{2}$, as shown in the complete solutio

**For 'd'**

We now eliminate all terms from Equation (1) except for the coefficients of x. The dwandwa-yoga of $y^2$ here is $2ab = 2 \times 0 \times 0$, and from Column II we have,

$$2\,ab + 6\,d = 0$$

$$d = 0$$

**For 'e'** ( Column III )

The dwandwa-yoga is now $2ac + b^2$, and we have

$$2 \times 0 \times \tfrac{1}{2} + 0^2 + 12e = 2$$

we write 2 above 12e, and 1/6 above e.

**For 'f'** ( Column IV)

Since the coefficient of $x^3$ in Equation (1) is zero, the ter in y'' equals minus the dwanda-yoga term, i . e. $-2ad-2bc$, which is zero; hence f = 0. And so the solution continues. Had y (0) not been zero there would have been two solutions. As it is, the two solutions coincide.

The next example has two solutions, both of which are given. The boundary conditions are given at x = 1, and so a series expansion in powers of ( x - 1 ) is used.

### 11.1.3. Illustration 3 :

Solve $2y + (y')^2 + 3y'' = 2\exp(x) + \exp(2x)$......(1)

given $y(0) = 2$, $y'(0) = 1$

Here the presence of $(y')^2$ might lead us to expect two solutions, but being given $y'(0) = 1$, there is only one solution. We first expand the right-hand side of Equation (1) as a power series in x :

$$2y + (y')^2 + 3y'' = 3 + 4x + 3x^2 + \frac{10}{3!}x^3 + \frac{18}{4!}x^4 + \frac{34}{5!}x^5 + \frac{66}{6!}x^6 + \frac{130}{7!} + \ldots$$

$$\text{Let } y = \overset{2}{A} + \overset{1}{B}x + \overset{0}{C}x^2 + \overset{\frac{1}{9}}{D}x^3 + \overset{\frac{7}{108}}{E}x^4 + Fx^5 + Gx^6 + Hx^7 + \ldots\ldots.$$

$$\frac{7}{24} = 0.3210\bar{1}$$

$$y' = \overset{1}{B} + 2\overset{0}{C}x + 3\overset{\frac{1}{3}}{D}x^2 + 4\overset{\frac{7}{24}}{E}x^3 + 5Fx^4 + 6Gx^5 + 7Hx^6 + 8Ix^7 + \ldots$$

$$y'' = 2\overset{0}{C} + 6\overset{\frac{2}{3}}{D}x + 12\overset{\frac{7}{9}}{E}x^2 + 20Fx^3 + 30Gx^4 + 42Hx^5 + 56Ix^6 + \ldots$$

I.e. the solution is :

$$\underline{y = 2 + x + 0.ix^3 + 0.064814815x^4 + \ldots\ldots}$$

### 11.1.4. Illustration 4 :-

Solve $y^{\frac{1}{2}}y' + (y')^2 = 6(2+x)^2$......... (1)

given $y(0) = 4$ .

We make use of the vedic method for finding the square root of a polynomial or power series. This is essentially the same as that for finding the square root of a number in decimal form -

Initially we assume a power series for y, and allow the successive terms of $y^{\frac{1}{2}}$ to unfold above it, the first derivative being placed below. Since $y(0) = 4$, a = 4 and the first term for $y^{\frac{1}{2}}$ is + 2 ( or - 2, leading to another solution ).

The solution proceeds :

$$y^{\frac{1}{2}} = +2 + \ldots\ldots$$

$$y = \overset{4}{a} + bx + cx^2 + dx^3 + ex^4 + fx^5 + \ldots\ldots$$

$$y' = b + 2cx + 3dx^2 + 4ex^3 + 5fx^4 + 6gx^5 + \ldots\ldots$$

Now Equation (1) can be rewritten,

$y'(y^{\frac{1}{2}} + y') = 24 + 24x + 6x^2$..............(2)

Considering only absolute terms, we have,

$$b(2+b) = 24$$

i.e. we seek two numbers differing by 2, whose product is 24. clearly 4 & 6 or -4 & -6 fit the bill ( hence a doubling up of the number of solutions ). Using 4 & 6, we have b = 4.

The solution continues:

$$y^{\frac{1}{2}} = 2 + 1x + \ldots\ldots\ldots \qquad \text{Row 1}$$

$$y = \overset{4}{a} + \overset{4}{bx} + \ldots\ldots\ldots \qquad \text{Row 2}$$

$$y' = \overset{4}{b} + \overset{2}{2}\,cx + 3dx^2 + \ldots \qquad \text{Row 3}$$

We are now ready to consider the coefficient of x, in Equation (2). The left-hand side of Equation (2) is satisfied by ( mentally ) adding Row 3 to Row 1, and multiplying cross-wise by Row 3. i. e.

$$y^{\frac{1}{2}} + y' = 6 + ( 1 + 2c ) x$$

$$y' = 4 + 2cx + \ldots\ldots\ldots$$

the coefficient of x is ( 1 +2c ) x 4 + 6 x 2c. And this must equal 24 ( from the right-hand side of Equation (2).

$$2c = ( 24 - 3\times4 ) \div 6 = 2,$$

$$\underline{c = 1}$$

Thus the solution so far is

$$y^{\frac{1}{2}} = 2 + 1x + 0x^2 + \ldots\ldots$$

$$y = \overset{4}{a} + \overset{4}{bx} + \overset{1}{cx^2} + \ldots\ldots$$

$$y' = \overset{4}{b} + \overset{2}{2cx} + 3dx^2 + \ldots\ldots$$

For on continuing the square-rooting process, the next coefficient ( that of $x^2$ ) is zero.

Considering now the coefficients of $x^2$ in Equation (2) we have,

$$y^{\frac{1}{2}} + y' \text{ gives } 6 + 3x + 3dx^2$$

$$\& \quad y' \text{ gives } 4 + 2x + 3dx^2$$

Hence, proceeding vertically and crosswise, the coefficient of $x^2$ comes to $6(3d) + 3x2 + 4x(3d) = 6$.

where the right-hand-side '6' is the coefficient of $x^2$ on the right in Equation (2,. Hence $d = 0$.

Continuing in this way we find that

$$y = 4 + 4x + x^2$$

$$= (2 + x)^2 \text{ exactly.}$$

This is one of four solutions.

## 11.2. Transcendental Equations

A special case of equations such as those presented so far is one containing no differentials, but simple terms in y, and functions of y, and functions of x. This is a type of transcendental equation, differing from those of Chapter 11 of vertical & crosswise in having two variables, one being considered to be dependent upon the other.

### 11.2.1. Illustration 5:-

$$\text{Solve} \quad y - 1ny = 1 - x^2, \qquad (1)$$

$$\text{Where} \quad y = y(x)$$

Assume that

$$y = a + bx + cx^2 + dx^3 + ex^4 + \ldots\ldots\ldots\ldots \qquad (2)$$

Substituting in Equation (1) and differentiating w.r.t. x :

$$\frac{\overset{0}{b} + \overset{-1}{2c}x + \overset{0}{3d}x^2 + \overset{\frac{1}{4}}{4e}x^3 + \overset{0}{5f}x^4 + \overset{\frac{1}{32}}{6g}x^5}{\underset{1}{a} + \underset{0}{b}x + \underset{-\frac{1}{2}}{c}x^2 + \underset{0}{d}x^3 + \underset{\frac{1}{16}}{e}x^4 + \underset{0}{f}x^5 + \underset{192}{g}x^6}$$

$$= \underset{0}{-b} - (2 + \underset{-1}{2c})x - \underset{0}{3d}x^2 - \underset{-\frac{1}{4}}{4e}x^3 - \underset{0}{5f}x^4 - \underset{\frac{1}{32}}{6g}x^5 - \ldots\ldots \qquad (3)$$

| STEPS | Equation Used |
|---|---|
| By observation, $y(0) = 1$ is a solution * | (1) |
| $a = 1$ | (2) |
| Equate coefficients of unity: $b = 0$ | (3) |
| " " " $x$ : $4c = -2$ | (3) |
| " " " $x^2$ : $3d = 0$ | (3) |
| " " " $x^3$ : $8e = \frac{1}{2}$ | (3) |
| " " " $x^4$ : $10f = 0$ | (3) |
| " " " $x^5$ : $12g = \frac{1}{16}$ | (3) |

A solution is :

$$y = 1 - \tfrac{1}{2}x^2 + \frac{1}{16}x^4 + \frac{1}{192}x^6 + \ldots\ldots\ldots\ldots$$

* This is the only real solution of $y - \ln y = 1$

1. $y' + 2y + b = 4x + 16$ $(y = 4 + 2x)$
2. $y' + \left(\frac{dy}{dx}\right)^2 = 676x^4 + 624x^3 + 378x^2 + 108x + 20$ $(y = 2 + 4x + 6x^2 + 13x^3)$
3. $y'' + y' - y = -10x^4 + 38x^3 + 126x^2 + 8x + 4$ $(y = 4x + 2x^3 + 10x^4)$
4. $y' - 3y'' + 6 = 4 - 14x - 32x' - 24x^3$ $(y = (1 + 2x)^3)$
5. $y'' + 2y' + 6y = 14x + 4x$ $(y = 1 + 4x)$
6. $2y + 3y' + 6y^2 = 5\exp(x) + 6\exp(2x)$ $(y = \exp(x))$
7. $4y' + 7y + 2 = 2 + 23\exp 4x - 11\exp x$ $(y = \exp(4x) - \exp(x))$
8. $\int_0^x y\,dx + 3\int_0^x y\,dx = 16 + 6x^2$ $(y = 4 + 3x)$
9. $\int_0^x\int_0^x y^2\,dx + \int_0^x y\,dx = 2x + \frac{7}{2}x^2 + 2x^3 + \frac{3}{4}x^4$ $(y = 3x + 2)$

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# वैदिक गणित अध्ययन

## Tutorial 1

## VEDIC BASE SYSTEM

1. Choose the appropriate base for the given nos. and write them in the Vedic Base System style.

(i) 10005 (ii) 999876 (iii) 8

(iv) 100 (v) 97 (vi) 123

(vii) 1345 (viii) 872 (ix) 1000001

(x) 999

2. Choose appropriate, base, sub base wherever required, and write in the Vedic style.

(i) 52 (ii) 49989 (iii) 899

(iv) 256 (v) 3003 (vi) 24937

(vii) 9827 (viii) 2700 (ix) 287

(x) 507

Note; These problems are taken for taking the submultiples of 1/2, 2, 3, 4, 5, 1/4 etc.

3. Write 232 taking (i) 100 as base and 200 as sub-base
(ii) 1000 " " 200 "
(iii) 1000 " " 250 "

Note: This problem illustrates that one no. can be represented by using different bases and sub bases.

4. Check the errors

| No. | Base | Sub-base | Submultiple | Vedic style |
|---|---|---|---|---|
| (i) 256 | 1000 | 250 | 2.5 | 250+006 |
| (ii) 303 | 100 | 300 | 3 | 303-3 |
| (iii) 999 | 1000 | x | x | 1000-1 |
| (iv) 9996 | 100 | x | x | 9996-0004 |
| (v) 196 | 100 | 200 | 2 | 196 -04 |
| (vi) 50016 | 1000000 | 50000 | 1/2 | 50016-16 |

In problem (i) of Question 4 it appears that 256 should be represented as 250+6.

Also we can think of a submultiple of 2.5, which quite inconvenient and hence not used.

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5. Find out the base and correct number in the following and also write the sub-base wherever necessary.

| Vedic style of writing | Deviation | Base | Correct Number |
|---|---|---|---|
| 19 - 1 | - 1 | | |
| 98 - 2 | - 2 | | |
| 105 + 5 | 5 | | |
| 198 - 2 | - 2 | | |
| 178 - 22 | -22 | | |
| 4955 - 45 | -45 | | |
| 225 + 25 | 25 | | |

# TUTORIAL 2
## (Vinculum)

1. Convert into Vinculum form

(i) 8453 (ii) 9871565

(iii) 84096785125 (iv) 8473609

(v) 999999 (vi) 8395654

Note: Problem (ii) tells us that to get Vinculum form in one step, if 5 precedes digits greater than it, convert it also to Vinculum form.

2. Normalize the following and check it by Vedic check method.

(i) $12\bar{3}\bar{4}5\bar{1}$ (ii) $5\bar{3}00\bar{1}\bar{2}$

(iii) $42\bar{1}0\bar{2}03\bar{4}0\bar{1}$ (iv) $5\bar{3}\bar{2}\bar{1}3\bar{4}0$

(v) $\bar{4}5\bar{4}2\bar{1}0\bar{2}3\bar{3}$

Notes: 1. Problems (ii) & (iii) require us to put 0 as $\bar{0}$ if they precede the bar digits.

2. The number in part (v) is negative.

3. What is Vinculum approach? Write the numbers in Vinculum form and specify the critical points in the problems.

(i) 98 (ii) 192 (iii) 1928

(iv) 193847 (v) 4070809 (vi) 999876

(vii) 7005809

IIB

# Tutorial 3

## TABLES

1. Write the operators and write the table upto 22nd level. Check at 3rd, 7th, 9th, 13th, 27th, 19th level by Vedic style.

(i) 12    (ii) 125    (iii) 124321

(iv) 8891 ⁞ Note: It's convenient to write the operators
(v) 9968 ⁞ by just converting into Vinculum form.

(vi) 10034    (vii) 832    (viii) 89

(ix) 1211

2. Find out the table of the number by Nikhilam indicating the base and sub-base upto 10th place.

(i) 98    (ii) 63    (iii) 155

(iv) 99    (v) 1981    (vi) 1986

(vii) 7777

3. Define the word operator. In the following numbers write the operator and make the table upto 15th place.

(i) 99    (ii) 105    (iii) 117

(iv) 198    (v) 78    (vi) 5678

(vii) 9888    (viii) 787    (ix) 9973

Pt-II-T1

## TUTORIAL - 1

## SQUARING - URDHAVA AND YAVADUNAM SUTRA

Q.1. Find the square of the following numbers using Urdhava sutra.

(i) $227^2$ (ii) $425^2$ (iii) $819^2$ (iv) $7336^2$

(v) $4987^2$ (vi) $67821^2$ (vii) $92416^2$

Q.2 Find the square of the following numbers using Yavadunam sutra.

(i) $9992^2$

(ii) $10006^2$

(iii) $100019^2$

(iv) $11102^2$

(v) $9989^2$

(vi) $99997^2$

(vii) $11115^2$

Q.3 Find the Square of the following numbers using Anurupena Sub sutr

(i) $46^2$ (ii) $252^2$ (iii) $411^2$

(iv) $689^2$ (v) $428^2$ (vi) $559^2$

(vii) $4235^2$

Q.4: Check the Solutions of Q 1, 2 and 3 using Vedic method.

Q. 5 Convert the above numbers to Vinculum form and then square them using urdhava sutra.

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T-11.1

## TUTORIAL 11

Q.1 Solve the following multiplications by using different combinations of base and working base. Identify the most efficient combination of base and the working base in each of the case:

(i) 42 x 42 (ii) 47 x 47 (iii) 52 x 54
(iv) 41 x 45 (v) 56 x 58 (vi) 43 x 44
(vii) 251 x 255 (viii) 31 x 47 (ix) 221 x 212
(x) 23 x 16 (xi) 13 x 31 (xii) 4782 x 4997
(xiii) 524 x 495 (xiv) 281 x 594 (xv) 497 x 39
(xvi) 368 x 511 (xvii) 512 x 522 (xviii) 675 x 498
(xix) 3989 x 5002 (xx) 72 x 9989 (xxi) 123 x 605
(xxii) 102 x 9867

Q.2 Discuss the use of Subsutra Anurupyena.

Q.3 Solve the following

(i) 11 x 13 (ii) 10001 x 10749 (to prefix zero's to get the ans.)
(iii) 8 x 6 (iv) 96 x 98 (v) 989 x 986
(vi) 13 x 9 (vii) 1003 x 998
(In vi and vii) we get Vinculum portion)
(viii) 253 x 242 (to use Anurupene)
(ix) 42 x 56 (x) 17 x 16 (xi) 33 x 31
(xii) 692 x 608 (By converting them to Vinculum form)
(xiii) 889 x 811 (xiv) 19994 x 10006
(xv) 109 x 1593281 (To shift the digits to left)
(xvi) 672 x 592

Check the correctness of the result by Vedic style.

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Pt II - T2

## TUTORIAL - 2

### SQUARING - DWANDWA YOGA

Q.1 Find the squares of the following numbers.

(i) $43^2$ (ii) $236^2$ (iii) $3697^2$

(iv) $567^2$ (v) $2345^2$ (vi) $14679^2$

(vii) $268578^2$ (viii) $370415392^2$ (ix) $432656^2$

Q.2. Find the squares of the following numbers using Vin culum approach

(i) $987^2$ (ii) $2689^2$ (iii) $68902^2$

(iv) $56326985^2$ (v) $3296586^2$

Q.3. Square the following numbers
(i) $321.21^2$ (ii) $127.38^2$ (iii) $24.279^2$

Q. 4. Square the following numbers
(i) $0.231^2$ (ii) $0.0376^2$ (iii) $.00245^2$

Q.5. Check the results of Q.1 to 0.4.

HINTS. (i) Q3 & Q.4, use the normal approach for fixing the decimal point.

(ii) Approximately round off the number and then fix the decimal point. Say for $.31^2 = .30^2$ ( approx.)

since $30^2 = 900$ : $302^2 = .0900$

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# TUTORIAL - 3

## SQUARE ROOT

Q.1. Find the square root of the following numbers.

(i) 1552542907 upto 3 decimal places

(ii) 753128 upto 4 decimal places

(iii) 7410756 upto 2 decimal places

(iv) 8009240270 upto 3 decimal places

(v) 1882405 upto 4 decimal places

(vi) 738915489

(vii) 25745476

(viii) 45319824

(ix) 984519837 Using Vinculum

(x) 92945241 using Vinculum

(xi) 1360996 using Vinculum.

Q.2. Find the square root of the following numbers upto 4 decimal places.

(i) 524.134

(ii) 64676.23

(iii) 2364.4502

(iv) 567.3498 using Vinculum approach

(v) 89.24678 using Vinculum approach

Q.3 Check the square root of the exact numbers.

# TUTORIAL

## CHAPTER 24

## OPERATION OF VINCULUM NUMBERS - PART I

Q. 1 Perform the addition of following Vinculum numbers and then simplify these:

i) $\begin{array}{r} 1\,\overline{2} \\ \underline{3\,\overline{4}} \end{array}$ (ii) $\begin{array}{r} 5\,\overline{2}\,\overline{4} \\ \underline{\overline{3}\,4\,3} \end{array}$ (iii) $\begin{array}{r} 5\,9\,2\,\overline{3} \\ \underline{\overline{3}\,8\,\overline{1}\,2} \end{array}$ (iv) $\begin{array}{r} 2\,\overline{3}\,5\,\overline{1}\,2 \\ \underline{4\,\overline{1}\,3\,\overline{1}\,2} \end{array}$

v) $\begin{array}{r} \overline{1}\,\overline{2}\,\overline{3}\,\overline{4}\,5 \\ \underline{3\,0\,\overline{1}\,8\,\overline{1}} \end{array}$ (vi) $\begin{array}{r} 1\,3\,\overline{4}\,5\,\overline{1}\,2 \\ \underline{3\,\overline{4}\,5\,\overline{1}\,2\,\overline{3}} \end{array}$ (vii) $\begin{array}{r} \overline{1}\,3\,\overline{2}\,2\,\overline{3}\,1\,\overline{4} \\ \underline{4\,\overline{1}\,3\,\overline{3}\,4\,\overline{2}\,1} \end{array}$

Q.2 Perfform the subtraction of following Vinculum numbers and then simplify the result with appropriate sign:

i) $\begin{array}{r} 5\,\overline{1}\,\overline{2} \\ \underline{-1\,\overline{3}\,1} \end{array}$ (ii) $\begin{array}{r} 1\,\overline{2}\,\overline{3}\,\overline{4} \\ -\ \underline{\overline{3}\,\overline{1}\,2\,1} \end{array}$ (iii) $\begin{array}{r} 8\,\overline{1}\,\overline{2}\,\overline{4}\,1 \\ -\ \underline{\overline{1}\,5\,\overline{2}\,1\,\overline{3}} \end{array}$

iv) $\begin{array}{r} 0\,1\,2\,3\,4\,5\,6 \\ -\,\underline{6\,\overline{2}\,\overline{3}\,\overline{4}\,\overline{5}\,\overline{4}\,\overline{1}} \end{array}$ (v) $\begin{array}{r} 2\,\overline{3}\,5\,1\,0\,1 \\ \underline{-\,4\,\overline{4}\,1\,\overline{2}\,1\,0} \end{array}$

Q. 3 Perform following multiplication of Vinculum numbers:

i) $7 \times \overline{2}$ (ii) $\overline{4} \times 8$ (iii) $\overline{3} \times \overline{4}$ (iv) $2 \times \overline{3}$

v) $8 \times \overline{4}$ (vi) $\overline{2} \times \overline{7}$ (vii) $1\overline{1} \times 3$ (viii) $1\overline{2} \times \overline{4}$

ix) $2\overline{3} \times 2$

Q.4 Which of the following is true or false? Correct the wrong solutions:

i) $\begin{array}{r} 3\,\overline{7}\,2 \\ \underline{+1\,\overline{3}\,\overline{1}} \\ \underline{5\,0\,1} \end{array}$ (ii) $\begin{array}{r} 3\,5\,\overline{5} \\ \underline{+\overline{3}\,\overline{4}\,5} \\ \underline{0\,0\,0} \end{array}$ (iii) $\begin{array}{r} 2\,3\,4 \\ +\,\underline{1\,\overline{3}\,\overline{4}} \\ \underline{1\,0\,0} \end{array}$

iv) $\begin{array}{r} 2\,7\,5 \\ -\underline{12\overline{2}\,3} \\ \underline{1\,9\,2} \end{array}$ (v) $\overline{3} \times \overline{7} = 21$ (vi) $2 \times \overline{4} = \overline{1}2$

vii) $1\,\overline{2}\,\overline{1} \times 3 = 237$

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# वैदिक गणित अध्ययन

## CHAPTER 25

## OPERATION OF VINCULUM NUMBERS - PART II

Q.1 Perform the subtraction of following vinculum numbers and then simplify these:

i) $4\,\bar{6}\,5\,\bar{5}$
$-3\,6\,2\,7$

(ii) $6\,\bar{6}\,3\,6\,\bar{9}$
$-3\,\bar{5}\,\bar{8}\,\bar{7}\,4$

(iii) $4\,2\,\bar{7}$
$-\bar{6}\,\bar{5}\,5$

iiv) $3\,0\,\bar{1}\,8\,\bar{1}$
$-\bar{1}\,\bar{2}\,\bar{5}\,\bar{4}\,5$

(v) $5\,4\,5\,3\,4$
$-3\,\bar{5}\,\bar{5}\,4\,\bar{7}$

(vi) $6\,5\,4\,3$
$-5\,\bar{4}\,\bar{6}\,\bar{3}$

vii) $3\,0\,\bar{7}\,\bar{4}\,\bar{2}$
$-2\,0\,\bar{5}\,3\,5$

Q.2 Perform the addition of the following Vinculum numbers and then simplify the result.

i) $5\,\bar{4}\,\bar{5}$
$+\bar{4}\,4\,5$

(ii) $3\,\bar{5}\,4\,\bar{6}$
$+\bar{2}\,\bar{5}\,6\,\bar{4}$

(iii) $6\,\bar{1}\,\bar{4}\,\bar{3}\,7$
$+\bar{4}\,2\,\bar{5}\,\bar{8}\,3$

Note: In all these examples (i), (ii), (iii) the lower number is essentially a negative number.

iv) $7\,\bar{2}\,\bar{3}\,\overset{6}{8}\,5$
$+2\,3\,4\,\bar{5}\,5$

(v) $4\,2\,5\,5\,\bar{2}\,\bar{5}\,\bar{6}$
$+2\,\bar{6}\,4\,\bar{2}\,3\,4\,\bar{4}$

Q.3 Check the following solutions and correct the wrong solutions

i) $3\,4\,\bar{5}\,\bar{5}$
$-3\,3\,\bar{5}\,4$
$1\,1\,0\,0$

(ii) $2\,3\,\bar{4}\,\bar{5}\,\bar{1}$
$-1\,2\,\bar{5}\,\bar{5}\,\bar{3}$
$1\,1\,0\,0\,4$

(iii) $3\,0\,\bar{2}\,\bar{3}$
$-1\,5\,2\,\bar{4}$
$2\,\bar{3}\,0\,1$

iv) $1\,3\,4\,\bar{4}\,\bar{3}$
$+2\,\bar{5}\,\bar{4}\,3$

$2\,3\,\bar{1}\,\bar{4}$
$\bar{4}\,4\,\bar{6}$
$4\,3\,2\,0$

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## TUTORIAL-CHAPTER 26

## MIXED ADDITION AND SUBTRACTION

I, Simplify given set of numbers as indicated below:

(i) 317 + 512 - 216 + 412 + 213 - 467 + 219

(ii) 3158 + 8196 - 816 + 8195 + 2163 - 956

(iii) 21893 + 8517 - 21385 - 8168 + 26595 - 31493 - 29617 +8197

(iv) 6156 - 8953 + 41315 + 21689 - 5123 - 6895 + 81963 - 67314

(v) 319 + 21589 - 15317 + 2168 - 539 + 81963 - 41957 - 213
+ 156 - 181

(vi) 645 - 346 + 2196 + 3245 - 899 - 767 + 392

(vii) 987 - 1994 - 2386 + 4578 - 2488 + 999 - 88
(Using Nikhilam Sutra)

(viii) 2456 + 399 + 645 - 245 + 482 - 420 - 299

(ix) 999 - 888 + 777 - 666 + 555 - 444

(x) 425 + 524 - 646 + 864 - 22 + 111 - 989

(xi) 517 + 480 - 395 - 267 + 1199 - 0467

# TUTORIAL-27

## CHECKING OF VINCULUM NUMBERS

(1) Apply vedic check in answers arrived at in Tutorial 26.

(2) Apply vedic check method in respect of all the problems corresponding to Chapter 24 and 25.

(3) Write True/False for the following. Use Vedic check method.

(i) $4597456 = 45135\overline{4}\overline{4}$

(ii) $98976423 = 0\,\overline{1}\,\overline{1}\,\overline{3}\,\overline{4}\,4\,2\,3$

(iii)
$$\begin{array}{rr} & 3\;4\;\overline{5}\;\overline{6} \\ + & \overline{2}\;5\;\overline{4}\;\overline{3} \\ \hline & 5\;9\;\overline{8}\;\overline{9} \\ \hline \end{array}$$

(iv)
$$\begin{array}{rr} & 5\;4\;6\;7 \\ - & 2\;\overline{4}\;\overline{6}\;\overline{8} \\ \hline & 3\;0\;0\;5 \\ \hline \end{array}$$

(v)
$$\begin{array}{rr} & 9\;9\;9 \\ - & \overline{9}\;\overline{9}\;\overline{9} \\ \hline & 1\;0\;0 \\ \hline \end{array}$$

(vi)
$$\begin{array}{r} 5\;4\;\overline{3}\;7 \\ 2\;\overline{4}\;5\;6 \\ 3\;\overline{5}\;\overline{5}\;6 \\ \hline 1\;0\;\overline{5}\;\overline{5}\;6 \\ \hline \end{array}$$

(vii)
$$\begin{array}{rr} & 5\;4\;\overline{3}\;6 \\ - & \overline{2}\;\overline{7}\;8 \\ \hline & 1\;3\;1\;8 \\ \hline \end{array}$$

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## TUTORIAL 13

## GENERAL FORMULA FOR MULTIPLICATION

1. Solve the problems of T10 and T12 by the general method, alongwith other applicable methods. Compare and find out the most convenient sutra in each case.
Solve Q.2 and 3 using URDHAVA method.

2. i) 21 x 63 (ii) 27 x 67 (iii) 33 x 73
iv) 133 x 179 (v) 141 x 281 (vi) 371 x 433
vii) 1278x 9034 (viii) 12078x 10304 (ix) $378^2$
x) 6471x 6012 (xi) 81265x 32112

3. i) 89588 x 787 (ii) 396 x 914
iii) 8858 x 78 (iv) 6548 x 787
v) 432008x 1212 (vi) 58010 x 7017
vii) 9238 x 1010101
viii) 880001 x 061010

4. Solve following using viniculum approach by general Sutra:-
i) 787 x 391 (ii) 282 x 132
iii) 181 x 98 (iv) 271 x 87
v) 591 x 187 (vi) 483 x 179
vii) 1892 x 197 (viii) 9867 x 918
ix) 90908x81826 (x) 81739 x 9019

5. What is the difficulty you are facing in these problems. Are you finding any other method for its simplification. If yes then solve your problem by that method.

6. Solve the problems by the help of Nikhilam and Urdhava both and compare
799 x 398, 5439 x 4569, 177 x 165
6789 x 9876

## TUTORIAL 14

## MULTIPLICATION CHECK

1. Check your answers of Q.2 of Tutorial 13 by this Vedic Approach of Multiplication check.
2. Are the following multiplication correct. Write True/False. Use multiplication check:

i) 96 x 98 = 9408 (ii) 879 x 998 = 886082

iii) 1016 x 1006 = 1122096

iv) 1033 x 977 = 1029901

v) 87965 x 99988 = 8796324070

vi) 128 x 672 = 86016 (vii) 635 x 502 = 318740

viii) $989^2$ = 987121 (ix) $775^2$ = 601625

x) 1021 x 2103= 2147163 (xi) 582 x 231 = 134542

xii) 87265 x 32117 = 2802690005

xiii) 817 x 332 = 271244

xiv)
$$\begin{array}{r} 1234567809 \\ \times\ 9999999999 \\ \hline 1234567808/8765432191 \\ \hline \end{array}$$

xv)
$$\begin{array}{r} 999999997 \\ \times\ 999999981 \\ \hline 999999878/000000057 \\ \hline \end{array}$$

## TUTORIAL - CHAPTER 20-21

## COMPOUND MULTIPLICATION

1. Compute following -

(i) 3'8"x2'1" (ii) 1 yd 2'x5 yd 1'

(iii) 2yd 2'3" x 3yd 1'x7"

Apply Vedic Check method in all the above cases.

2. Compute volume in case of following parallelopieøeds whose dimensions are -

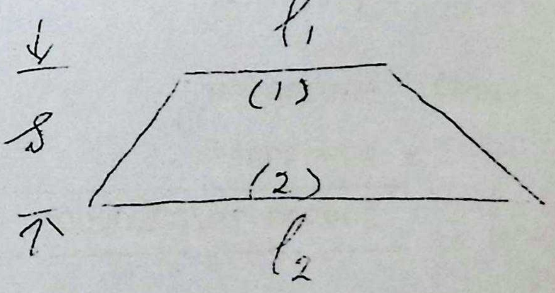
(i) 2'5", 2'2" , 6' 8"

(ii) 4' 18", 3' 9", 2' 8"

Answer should be in term of cubic ft and cubic inches only. Apply the check.

3. Compute the area of trapizium whose lengths of parallel sides and their separation is as follows -

| | Length | | Separation |
|---|---|---|---|
| | (1) | (2) | 'S' |
| (i) | 3'6" | 2' 7" | 1' 3" |
| (ii) | 5'8" | 3' 11" | 2' 6" |



# TUTORIAL-33

## DIVISION CHECK

1. Check your answers of the followings. Use the Vedic checking method for division.

   (i) Problem I of tutorial (29)

   (ii) Problem I, II and III of Tutorial (30)

   (iii) Problem I and III of Tutorial (32)

   (iv) Problem I and II of Tutorial 31 and 34.

2. Write True/False for the following divisions. Use Vedic Division check.

   (i) 9/12000212 = 133356/8 Q/R

   (ii) 8888/101020 = 12/3252

   (iii) 88987/1030007 = 10/40237

   (iv) 23/1011 = 43/22

   (v) 7989/ 10102 = 1/2913

   (vi) $x^2-x-\underline{1}/\ x^4-x^3+x^2+3x+5$

   $\therefore\ Q = x^2 + 12\ ,\ R = 6x + 7$

   (vii) $2x-\underline{4}/\ -4x^3-7x^2+9x-12$

   $\therefore\ Q = -2x^2 - 7\frac{1}{2}x - 13\frac{1}{2}$

   $R = -54$

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T-29

## TUTORIAL-29

## DIVISION (BY NIKHILAM METHOD)

Q.1 Perform division by Nikhilam Method in the following cases

(i) 12483 ÷ 7988 (ii) 10015 ÷ 89

(iii) 12345 ÷ 8899 (iv) 124567 ÷ 882

(v) 210011 ÷ 8996 (vi) 1010108 ÷ 8988

(vii) 1020017 ÷ 98987

Q.2 (i) Divide 11000 by 88

(ii) Divide 1987 by 899

(iii) Divide 1294468 by 89997

(iv) Divide 11299251 by 99973

(v) Divide 2645678 by 7777

(vi) Divide 789645 by 8787

(vii) Divide 245654 by 9788

Q.3 (i) Divide 145672 by 99 upto 3 places of decimals.

(ii) Divide 1758011 by 887 upto 4 places of decimals.

(iii) Divide 101010 by 23 upto 2 places of decimals.

(iv) Divide 1213212 by 799 upto 3 places of decimals.

(v) Divide 11223311 by 8989 upto 2 places of decimals.

(vi) Divide 10010245 by 78779 upto 2 places of decimals.

(vii) Divide 250000 by 79 upto 3 places of decimals.

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T-31

# TUTORIAL-31

## ALGEBRAIC DIVISION

Q.1. Perform following division using paravartaya formula.

(i) $4x^3 - 7x^2 - 7x + 12$ by $2x-4$

(ii) $3x^2 - x - 5$ by $3x - 7$

(iii) $2x^5 - 9x^4 + 5x^3 + 16x^2 - 16x + 36$ by $2x^2-3x+1$

(iv) $x^4 + 4x^3 + 6x^2 + 4x + 1$ by $x^2+2x+1$

(v) $x^4 - 4x^2 + 12x - 9$ by $x^2 + 2x + 1$

(vi) $16x^4+36x^2 + 16x + 9$ by $4x^3 - 2x^2+2x-7$

(vii) $x^5+ x^3 - 7x^2 + 9$ by $x^3 - 2x^2 + 2x -3$

(viii) $21x^6+7x^5+15x^4+29x^3+x^2+15x+3$ by $3x^3+x^2+3$

Q.2 Write down the answer (i.e. the quotient and the remainder) in following cases by performing mental calculations using paravartaya formula.

(i) $x^3 + 7x^2+9x +11 \div x - 2$ ; Q = R =

(ii) $x^3 - x^2 -7x +3 \div x - 3$ ; Q = R =

(iii) $x^4 -3x^3+7x^2+5x+7 \div x - 4$ ; Q = R =

(iv) $-4x^3-7x^2+9x +12 \div x+1$ ; Q = R =

(v) $x^4-3x^3 +7x^2+5x+7 \div x+4$ ; Q = R =

(vi) $6x^4+13x^3+39x^2+37x+ 45 \div x^2-2x-9$; Q = R =

(vii) $2x^4-3x^3+3x - 2 \div x^2+ 1$; Q = R =

वेदात् सर्वं प्रसिद्ध्यति
वेद सम्पूर्ण ज्ञान के अनादि अनन्त स्रोत है।
**Vedas are the Eternal Source of All Knowledge**

## TUTORIAL-32

## STRAIGHT DIVISION

1. Perform division in following cases

(i) 48786 by 83
(ii) 60998 by 65
(iii) 1638467 by 138
(iv) 8956425 by 78
(v) 888888 by 46
(vi) 753142 by 38
(vii) [illegible]8 by 123.

2. (i) Divide 8475 by 65 upto 3 places of decimals.
(ii) Divide 750.2532 by 781 upto 4 places of decimals.
(iii) Divide 80.23 by 243 upto 5 places of decimals.
(iv) Divide 764256 by 1134 upto 2 places of decimals
(v) Divide 84246[illegible] by 246 upto 3 places of decimals.

3. Divide following by straight division method using Nikhilam method for division and dividend.

(i) 9168456 ÷ 8567
(ii) [illegible]24563949 ÷ [illegible]78
(iii) 784653 ÷ 649
(iv) [illegible]4652 ÷ 69

4. Divide following by straight division method using table for suitable divisor digit

(i) 642456 ÷ 59
(ii) 7456 ÷ 41
(iii) 524567 ÷ 55
(iv) 845 ÷ 89
(v) 7[illegible] ÷ 119

## CORRECTIONS

(A) Chapter 30, Illustration 14 on Page No.30.9

After the line using Anurupena we modify the quotient, Please read as

$\underline{2}/462/\bar{1}\ 40$

$231/\ \overline{60}$

$230/\ 198 - 60$

$230/138$

So $Q = 230$

$R = 138$

(B) Chapter 29. Illustration 22 on Page No.29.19 after line no.8, as given below

$/4\ 2\ 0\ 1$

$2\ 2\ 4\ /8$

$5\ 5\ /10\ 20$

$18\ /18\ 36\ 72$

$/34\ 34\ 68\ 136$

$1\ 1\ 1\ 2\ 5_1 8_3 4\ \ /7^4\ 9^2 14^0 13^7$

$1\ 1\ 1\ 2\ 7\ 1\ 4\ /\ 8\ 4\ 7\ 3\ 7$

$1\ 1\ 1\ 2\ 7\ 1\ 4\ /\ 8/\ 4\ 7\ 3\ 7$

$8\ 8\ 16\ 32$

$1\ 1\ 1\ 2\ 7\ 1\ 4/8/\ 1^2\ 1^5 1^9 3^9$

$1\ 1\ 1\ 2\ 7\ 1\ 4+8/1\ 3\ 7\ 2\ 9$

$1112722/13729$

$1\ 1\ 1\ 2\ 7\ 2\ 2/1/3\ 7\ 2\ 9$

$1\ 1\ 2\ 4$

$1\ 1\ 1\ 2\ 7\ 2\ 2/1/4\ 8\ 5\ 3$

$1\ 1\ 1\ 2\ 7\ 2\ 2 + 1/4\ 8\ 5\ 3$

$1112723/4853$

So

$Q = 1112723$

$R = 4853$

(c) Chapter 29 illustration 28 on Page 29.22 after the line 2 Please read as

$8\ 8\ 8\ 8\ 8\ 9 \div 1\ 2\ 4\ \bar{5}/\ \bar{4}\ \bar{3}\ \bar{2}\ \bar{1}\ \bar{1}\ 0$

## CORRECTIONS

Chapter 32 Page 32.4 item 32.3 line 2

a) Read as follows
we are actually dividing by using one digit

b) Chapter 32 Page 32.6 illustration 5. step 5
(ii) add the following

From the gross remainder we obtain the net remainder by subtracting the product of the last digit of Q and last flagdigit (i.e. 8 x 4 = 32) 32 from 295 (295 - 32 = 263). So 263 is the net Remainder.

Note:- The illustration can also be done by the another method. If we take the operator 82 and the flag digit only 4 than our work will be very easy and simple. For doing this first we make a table of 82 upto 9th level using the fast vedic method. So you try to solve the illustration by this method.

c) Chapter 32 Page 32.9 Illustration 7 step 5(i)
Please also read the discussion at item 32.4 (page 32.10)

d) Chapter 32 Page 32.10 Illustration 8
Please read the discussion at item 32.4 (Page 32.10).

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# आध्यात्मिक अध्ययन संस्था, रुड़की (आऽस)

## SPIRITUAL STUDY GROUP (SSG)

भारतवर्ष का पुनरुत्थान होगा। पर वह शारिरिक शक़्ति से नही, वरन् आत्मा का शक्ति द्वारा। वह उत्थान विनाश की ध्वजा लेकर नही, वरन् शांति और प्रेम की ध्वजा से.... मैं अपने सामने यह एक सजीव दृश्य अवश्य देख रहा हूं कि हमारी यह वृद्ध माता पुनः एक बार जागत होकर अपने सिंहासन पर नवयौवनपूर्ण और पूर्व की अपेक्षा अधिक महामहिमान्वित होकर विराजमान है। शाँति और आशीर्वाद के वचनो के साथ सारे संसार में उसके नाम की घोषणा कर दो।

तुम तो ईश्वर की संतान हो अमर आनंद के भागो हो, पवित्र और पूर्ण आत्मा हो। अतएव तुम कैसे अपने को जबरदस्ती दुर्बल कहते हो? उठो, साहसी बनो, वीर्यवान होओ।

केवल आध्यात्मिक ज्ञान ही ऐसा है जो हमारे दुःखो को सदा के लिए नष्ट कर सकता है।

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